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Irving P. Herman

Physics of the Human Body

Second Edition



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Physics of the Human Body

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*This book is dedicated to Daniel, Jonathan,
and Janet, to Sandy Koufax, and to the life of
Ron McNair*

Preface to the Second Edition

Near the end of one of the most famous speeches in American history, Abraham Lincoln made reference to “government of the people, by the people, for the people” [5]. In a similar vein, a text about physics and the human body could cover “physics of the body, by the body, and for the body,” but perhaps that would be too broad. Our main focus in this second edition is still the physics of the body, namely the physics of physiology, and the text is again largely organized by topics in physiology. There is still a strong emphasis on modeling physiology. As noted in the excellent book by Rideout [7], general modeling in physiology includes that of momentum (such as blood flow), mass (oxygen, carbon dioxide, and so on in the blood), energy (heat flow), and information (signals in nerves, hormones in the bloodstream) [7]. Models coupling multiple systems are also important. (After all, we are all strongly coupled multiple physiological systems, which are, of course, ultimately determined by physics.) We will still confine ourselves to the more physical and less chemical aspects of physiology modeling; chemical modeling is addressed well elsewhere [1, 2, 3, 4, 6, 7]. Within our focus of “physics of the body,” we have enhanced the discussion of the physics of disease, aging, motion, and the structure and function of muscles, and now explore how balance, metabolism, and the circulatory system change during pregnancy and birth. The physics of the cross-cutting lifecycle events of pregnancy, injuries, diseases and disorders, aging, and dying transcend the topics of any single chapter. They are now over-viewed in the new Appendix F; this appendix also describes how to locate discussions of these topics within the main text.

“Physics by the body” includes the physics of common locomotion and that during athletic events and collisions. These topics were addressed in several parts of the first edition, including many problems. They are now discussed in more detail in both the text and the problems. In particular, the physics of motion, kinesiology, and balance [8] are more directly discussed by the analysis of multi-segment modeling and the expanded treatment of the physics of sports. A discussion of human performance of ordinary tasks and at the extremes of athletic competition also transcends the topics of any single chapter and couples many

of them, especially those addressing statics, motion, metabolism, breathing, and blood circulation. To help unify the discussion of athletics, Appendix G has been added to link relevant topics in these chapters and to unify references to different sports activities.

“Physics for the body” includes the physics of measurement analysis and diagnosis and the physics of treatment, much of which is called medical physics. The first edition had some discussion of measurements and diagnosis and essentially none concerning treatment. The presentation of measurement analysis included several methods and a bit about measurement results. A broad overview of this topic is now presented in the new Appendix H, which includes a locator describing where physics analysis of the body is described in the text and problems. Aside from the new brief discussion in Appendix H, treatment is still not addressed in this edition.

This text can still be adapted for use in either lower- or upper-level undergraduate courses, as well as for some low-level graduate courses. It can serve the needs of a primary text for an entire course or as a secondary source, such as through the use of individual chapters. Appendices F, G, and H unite the cross-cutting themes presented in several chapters, and can be used to guide discussion of these topics.

This edition includes numerous additional problems that address the material in the text and explore it in much more detail. Many additional solutions are now provided at the end of the book. The author strongly recommends that you at least peruse the content of the related problems at the end of the chapter when you read any section of the text (even if you do not plan on solving them) because many essential approaches and ideas are covered only in the problems.

Several excellent texts on related topics were cited in the previous edition. In particular, we note again several of the outstanding books describing mathematical models of physiology [1, 2, 3, 4, 6, 7], which are for the most part at a level more advanced than that of this text. Also, of more general interest is the recent “The Story of the Human Body” by Daniel E. Lieberman, which discusses the evolution of the human body and includes extraordinary insights into the structure and function of the bodies of modern-day humans.

The author is indebted to Julio Fernandez, Glenn Fleisig, Joel Gersten, Daniel Herman, Jonathan Herman, Ronald Koder, Jeffrey Kysar, David Lerner, Kristin Myers, Herman Pontzer, and Alejandro Tello, and many others for their comments, suggestions, and insights as he prepared this second edition, to the numerous students, including Sarah Beck, who found errors in the first edition, and to those who helped in preparing the first edition. He would also like to thank the Columbia University Library system for speedy retrieval of reference books and articles (and for letting the author renew books many, many times).

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Irving P. Herman

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Preface to the First Edition

Physics explains everything from the beginning to the end of any complete description of the human body. Such a comprehensive discussion should begin with the basic structure of matter, as explained by quantum mechanics—the physics at small dimensions, and end with the mechanics of human motion, the energetics of metabolism, the fluid dynamics of blood flow through vessels, the mechanisms for speaking and hearing, and the optical imaging system we call the eye. All of required combinations of atoms to form the complex molecules and organs of organisms that live and reproduce can be explained by quantum mechanics; however, such explanations can get pretty complex. The fields of chemistry and biology have been developed, in part, to explain the gap between the extremes—the microphysics and macrophysics of organisms such as the human body.

This book focuses mostly on the macrophysics end of the human body. We will assume that atoms form molecules that form cells that form organs. We want to understand the physics of human organs and of humans themselves. We will apply and somewhat extend freshman-level physics to see how the body works. In addition to applying physical concepts to the body, we will try to understand the body from a viewpoint that is more numerical than is often adopted in biological and medical presentations.

One way to characterize this text is by saying what it is and what it is not. It is certainly about the physics of the human body. It is not about human anatomy, although we will need to use some basic anatomical concepts. It is not about human physiology, although it can be called a book about the physics of physiology. It is not a monograph in biomedical engineering per se, although about half of this volume concerns biomechanics, one important area in biomedical engineering. Medical physics is more closely related to health physics, the use of ionizing radiation, imaging, and instrumentation than to the macrophysics of the body. Biophysics concerns how physics can be used to study biology and focuses much more on the molecular basis and the cellular basis than will we (see Appendix E). One could say that the physics of the human body is synonymous with understanding the human machine.

Our goal is to understand physical issues concerning the human body, in part by solving problems to further this understanding. The focus is not at all on learning and memorizing medical terminology. Still some very basic concepts in anatomy and physiology will be introduced and used. Several of the many excellent general anatomy and physiology texts are cited at the end of the chapter [11, 16, 21, 22, 23, 24, 25, 26, 27, 29].

One theme that runs throughout this text is developing and then using simple and subsequently more refined models of the macrophysics of the human body [7, 13, 15]. Physicists tend to model concepts in as simple terms as possible at first. For example, to zero order a physicist would model a cow as a sphere. (This is sometimes used as part of a joke.) We will get a bit more complex here, but not much more. Another theme is to address issues in human biology quantitatively that are often addressed only qualitatively. The call for more quantitative thinking in physiology by Burton in *Physiology by Numbers* [5] is much appreciated by the author. In addition, we will present real physiological data and tie them with quantitative analysis and modeling.

If there is an applied force, energy, fluid flow, a light ray, an electric current, or an electric or magnetic field associated with the body, we will call it physics and we will analyze it. We will tend to avoid topics that delve into more chemistry and biology issues, but will briefly address physical chemistry issues involving concentration gradients and such, as they relate to fluid exchange in capillaries and conduction in nerves. Although we emphasize the physics of the body over the instrumentation used to make physical measurements on the body and probe body function, such instrumentation is addressed as needed.

Our intent is to use basic physics and not to teach it, particularly from scratch. Many chapters include a brief review of the physics principles needed in that chapter and subsequent chapters. Some topics are developed a bit further, and some even a bit further—and these are identified as advanced topics. More detailed overviews are given for topics seldom covered in detail during a two-semester physics course, such as fluids (Chap. 7), acoustics (Chap. 10), and optics (Chap. 11) and for areas used in several contexts, such as harmonic motion (Chap. 3). Some differential and integral calculus is used. (Partial differentiation is used sparingly, and mostly in sections labeled as advanced topics.) A brief review of the solutions to the simple differential equations used here is presented in Appendix C to help students with a limited background in calculus.

We will start with a comparison of medical- and physics-type terminology in Chap. 1. The first chapter also includes a discussion of the “standard” human and introduces the concept of scaling relations. We can group the topics in subsequent 11 chapters into four areas in human body physics. (1) In Chaps. 2–5, the mechanics of the static body (Chap. 2) and the body in motion (Chap. 3) are analyzed and are then linked to the mechanical properties of the materials of the body (Chap. 4) and the body’s motors: muscles (Chap. 5); these topics can be characterized as *Locomotion on Land*. (2) The second topic, *Energetics of the Body Metabolism*, is discussed in Chap. 6 and is needed to understand the discussions of body locomotion and function that precede and follow it. (3) Chapters 7–9 cover the *Locomotion of Humans in Fluids (other than on land) and the Motion of Fluids in*

Humans. Chapter 7 overviews the physics of fluids and addresses locomotion in water (swimming) and in air—above ground (at least, the prospect for human flying). Chapters 8 and 9 respectively cover the fluidics of blood (cardiovascular system) and air (respiratory system) in the body. (4) Chapter 10 explores the acoustics of sound waves in speaking and hearing. The optics of eyes and vision are investigated in Chap. 11. Basic electrical properties of the body are developed in Chap. 12, along with a brief description of the magnetic properties of the body. So these three chapters respectively address sound, electromagnetic, and electrical waves, which we can collectively call *Waves and Signals*. (The electromagnetic nature of light waves is not discussed in Chap. 11.)

Chapter 13 examines how the body automatically uses the basic engineering principle of feedback and control in regulating all aspects of function. The physics of sensation of three of the five senses are described: hearing, seeing, and touch—the last briefly in Chap. 2. Some connection is made between the physics of sensation, biochemistry of sensation, and perception (psychophysics) in Chap. 1. The sense of taste and smell are purely chemical, with little basis in physics (other than the chemistry of the molecular interactions in each being clear applications of physics), and are not covered—except for a brief discussion of the electrical properties of the taste and smell sensory neurons in Chap. 12. The emphasis throughout is on how physics can explain the functioning of the body under normal and unusual circumstances. We will concern ourselves with the human body with its common body coverings: footwear to minimize stress during movement (Chap. 4), clothes to regulate heat loss (Chap. 6), and corrective lenses to improve vision (Chap. 11).

The chapters are set more to address specific areas in physics rather than specific parts or systems in the body. It is difficult to construct chapters with clean divisions because different areas of physics are needed to understand many components of the body. For example, to understand the physics of the heart, you need to address its role in circulation (Chap. 8), the action of muscles (Chap. 5, which is more focused on skeletal muscle than the fairly similar cardiac muscle), and the electrical signals generated by the heart (Chap. 12).

This text concludes with five appendices. Appendix A overviews symbols and units, and references tables of units presented in the chapters. Appendix B lists the figures and tables that describe the main features of human anatomical and anthropometric information, which are used throughout this text. The types of differential equations used in the text are reviewed in Appendix C. These same differential equations are used throughout the text in mechanical, fluid flow, and electrical models; the connections between these models are made in Appendix D. Appendix E attempts to define the field of biophysics, and connects the contents of this text with this field.

This text has been developed from the author's lecture notes developed for the course *Physics of the Human Body*, which is a “professional-level” restricted elective course he developed taken mostly by first- and second-year undergraduates in the Columbia University Fu Foundation School of Engineering and Applied Science. This course was designed so it could be taken by all first-year students in

their second term (in conjunction with second-term physics and calculus). The author usually covers Chaps. 1–10 in some detail and Chaps. 11–13 in less detail in a full semester.

Courses at different levels, including mid-level and upper-level undergraduate courses, can be taught by purposely including or excluding more detailed and advanced topics in the text and problems. Depending of the level of desired depth, material in about half to all the chapters can be covered in one term.

This text can also be used as a companion volume in introductory physics courses, and assist premedical undergraduates in learning and reviewing physics. It can also serve as a text in introductory biomedical engineering or medical materials courses. Medical students interested in a more quantitative approach to physiology and those doing medical research may also appreciate the approaches adopted here.

Many problems are presented at the end of each chapter, ranging from simple to more advanced problems (the latter are denoted as such). Several problems have multiple parts, and only a few of these parts can be assigned. Answers to selected problems are given after the appendices.

Usually SI (MKS, m-kg-s) units are used; when more convenient, other metric units, including CGS (cm-g-s) units and mixed metric units are used. English FPS (ft-lb-s) units are sometimes purposely used to make a connection to the real world (at least in countries such as the USA and the UK). For example, it would be strange to hear a baseball announcer say, “This pitcher is really throwing some heat. The radar gun clocked his last pitch at 43.8 m/s (or 158 km/h),” as opposed to 98 mph. It would be stranger to hear a football (i.e., American football) announcer say, “They have first (down) and 9.144 to go,” meaning 9.144 m instead of 10 yd. Similarly, it would be strange to discuss the physics of the body in these sports, such as in throwing a baseball, in any but the usual units. Angles are given in radians, except when using degrees gives a more physical picture.

Several excellent texts cover material that overlaps topics covered here, each with a different focus. They are magnificent resources in their own right. *Physics of the Body* by Cameron, Skofronick, and Grant [6] spans most of the topics in this book and provides excellent physical insight. It is at a level of physics that is lower than that used here and derives and presents fewer of the equations necessary for a more rigorous treatment, but it provides a very good basic background in human physiology for nonexperts. In a way, the emphasis of *The Human Machine* by Alexander [2] coincides with ours, but, again, the explanations are more qualitative. The mode of physical thinking it presents is impressive. *Physics with Examples from Medicine and Biology* by Benedek and Villars [3] is a series of three comprehensive introductory physics texts in which excellent examples and problems have been chosen concerning the physics of the body. The authors have taken several fairly complex topics and have made them utterly understandable. Many other first-year general physics texts commonly used nowadays have several examples and chapter problems dealing with the body. *Intermediate Physics for Medicine and Biology* by Hobbie [14] is a more advanced text that emphasizes both physics and physical chemistry. *Medical Physics and Biomedical Engineering* by Brown et al. [4] is a bit more advanced and focuses also on classic areas in medical

physics, such as radioactivity and instrumentation. Many of the illustrative problems concerning human biology and related topics have been collected in the beautiful books: *Biomedical Applications of Introductory Physics* by Tuszynski and Dixon [28], *Physics in Biology and Medicine* by Davidovits [9], *Biophysics Problems: A Textbook with Answers* by Maróti, Berkes, and Tölgyesi [17], *Physics for the Biological Sciences: A Topical Approach to Biophysical Concepts* by Hallett, Stinson, and Speight [12], and *Topics in Classical Biophysics* by Metcalf [18]. Many of the issues in exercise physiology, such as the metabolism during sporting activities, are described in elementary terms in *Fox's Physiological Basis for Exercise and Sport* by Foss and Keteyian [10] and *Physiology of Sport and Exercise* by Wilmore and Costill [30]. *Basic Biomechanics of the Musculoskeletal System*, edited by Nordin and Frankel [20] is a comprehensive and clear overview of the biomechanics of structures, joints, and motion. The applications of physics at a more molecular and cellular level, more in the classical domain of biophysics, are described in *Biophysics: An Introduction*, by Cotterill [8] and *Biological Physics: Energy, Information* by Nelson [19]. The more general application of physics to animals is addressed in the exciting and very comprehensive book *Zoological Physics: Quantitative Models, Body Design, Actions and Physical Limitations in Animals* by Ahlborn [1]. All of these texts are highly recommended for more details. They, along with the anatomy and physiology texts cited earlier, have contributed to the preparation of this text.

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Contents

1	Terminology, the Standard Human, and Scaling	1
1.1	Anatomical Terminology	2
1.2	Motion in the Human Machine	5
1.3	The Standard Human	16
1.4	Scaling Relationships	23
1.4.1	Allometric Rules	24
1.4.2	Scaling in the Senses	26
1.5	Summary	27
	Problems	27
	References	37
2	Statics of the Body	39
2.1	Review of Forces, Torques, and Equilibrium	39
2.2	Statics: Motion in One Plane and Levers	42
2.3	Statics in the Body	46
2.3.1	The Lower Arm	46
2.3.2	Hip Problems	52
2.3.3	Statics of Other Synovial Joints	61
2.3.4	Lower Back Problems	69
2.3.5	Three-Force Rule	78
2.3.6	Multisegment Modeling	79
2.4	The Sense of Touch	81
2.5	Diversion into the Units of Force and Pressure	82
2.5.1	Force	82
2.5.2	Pressure	83
2.6	Summary	84
	Problems	85
	References	93
3	Motion	95
3.1	Kinematics and Musculature	98
3.2	Standing	105

3.2.1	Stability	105
3.2.2	Forces on the Feet	109
3.3	Walking	109
3.3.1	Kinematics	110
3.3.2	Muscular Action	112
3.3.3	Friction	113
3.3.4	Energetics	118
3.3.5	Review of Harmonic Motion, Pendulums, and Moments of Inertia	121
3.3.6	Ballistic (or Pendulum) Model of Walking	126
3.3.7	Inverted Pendulum Model	129
3.4	Running	130
3.4.1	Kinematics	130
3.4.2	Muscular Action	132
3.4.3	Energetics	133
3.4.4	Bouncing Ball/Pogo Stick Model	139
3.5	Jumping	140
3.5.1	Vertical Jump	141
3.5.2	Pole Vault	145
3.5.3	High Jump and Long Jump	147
3.6	Throwing a Ball	150
3.6.1	Comparing Throwing Motions	163
3.6.2	Power Generated During a Throw	164
3.7	Batting and Swinging Motions	165
3.8	Other Types of Motion	167
3.9	Multisegment Modeling (Advanced Topic)	170
3.10	Collisions of the Human Body	173
3.10.1	Kinematics of a Collision	173
3.10.2	Consequences of Collisions	175
3.10.3	Contact Sports	182
3.10.4	Hitting Balls	187
3.10.5	Running	191
3.10.6	Jumping	193
3.10.7	Chewing Food	193
3.11	Sustained Acceleration	194
3.12	Flight of the Ball	195
3.12.1	The Effect of Air Drag on Moving Balls and Humans	196
3.12.2	The Flight of a Spinning Ball	202
3.12.3	Bouncing, Rolling and Hitting Balls, including those with Spin	204
3.13	Summary	210
	Problems	210
	References	240

4	Mechanical Properties of the Body	247
4.1	Material Components of the Body	250
4.1.1	Bone	251
4.1.2	Ligaments and Tendons	253
4.1.3	Cartilage	254
4.2	Elastic Properties	255
4.2.1	Basic Stress–Strain Relationships	255
4.2.2	Other Stress–Strain Relations	258
4.2.3	Bone Shortening	259
4.2.4	Energy Storage in Elastic Media	260
4.3	Time-Independent Deviations in Hookean Materials	263
4.3.1	Non-Hookean Materials	272
4.4	Static Equilibrium of Deformable Bodies (Advanced Topic)	274
4.4.1	Bending of a Beam (or Bone)	279
4.5	Time-Dependent Deviations from Elastic Behavior: Viscoelasticity	283
4.5.1	Perfect Spring	287
4.5.2	Perfect Dashpot	287
4.5.3	Simple Viscoelastic Models	289
4.6	Viscoelasticity in Bone	297
4.7	Bone Fractures	299
4.7.1	Modes of Sudden Breaking of Bones	300
4.7.2	Stress Fractures (Advanced Topic)	307
4.8	Common Sports Injuries	311
4.9	Avoiding Fractures and Other Injuries: Materials for Helmets	314
4.10	Mechanical Properties of Food	317
4.11	Summary	319
	Problems	319
	References	326
5	Muscles	331
5.1	Skeletal Muscles in the Body	332
5.1.1	Types of Muscle Activity	335
5.2	The Structure of Muscles	336
5.3	Activating Muscles: Macroscopic View	341
5.3.1	Mechanical Model of the Active State of Muscles	344
5.4	Passive Muscles	350
5.5	Active/Tetanized Muscles: Microscopic View	351
5.5.1	Total Muscle Tension	352
5.5.2	Everyday Proof of the Limited Range of Useful Muscle Length	353
5.6	Hill Force–Velocity Curve	355
5.7	The Combined Muscle-Tendon System	362

5.8	The Sliding Filament Model: Nanoscopic View	363
5.8.1	Statistical Model of the Hill Force–Velocity Curve (Advanced Topic)	368
5.9	Titin	370
5.10	Coordination of Muscles	371
5.11	Muscle Strength and Evolution	374
5.11.1	Increasing Strength with Training	374
5.11.2	Muscle Evolution with Age	376
5.11.3	Muscle Fatigue	377
5.12	Summary	378
	Problems	378
	References	388
6	Metabolism: Energy, Heat, Work, and Power of the Body	393
6.1	Conservation of Energy and Heat Flow	394
6.2	Energy Content of Body Fuel	396
6.2.1	Metabolizable Energy	400
6.2.2	Energy Storage	402
6.3	Energy Storage Molecules	405
6.3.1	How ATP Is Produced and Used as an Energy Source	405
6.3.2	How ATP Is Actually Used by the Body	407
6.4	Metabolic Rates	415
6.4.1	Basal Metabolic Rate	415
6.4.2	Metabolic Rates During Common Activities	421
6.4.3	Weight Gain and Loss	438
6.5	Loss of Body Heat	442
6.5.1	Modes of Heat Loss	444
6.6	Body Temperature	459
6.7	Summary	466
	Problems	467
	References	485
7	Fluid Pressure, Fluid Flow in the Body, and Motion in Fluids	491
7.1	Characteristic Pressures in the Body	492
7.1.1	Definition and Units	492
7.1.2	Measuring Pressure	492
7.2	Basic Physics of Pressure and Flow of Fluids	494
7.2.1	Law of Laplace	495
7.2.2	Fluids in Motion	497
7.2.3	Equation of Continuity	498
7.2.4	Bernoulli’s Equation	499
7.2.5	Interactions Among the Flow Parameters	500
7.2.6	Resistance in Flow: Viscous Flow and Poiseuille’s Law	501

7.2.7	Compliance in Flow	509
7.2.8	Inertance in Flow	511
7.2.9	Flow Under Special Conditions	512
7.2.10	Flow of Objects in Fluids: Drag and Lift	513
7.3	Diffusion (Advanced Topic)	515
7.4	Pressure and Flow in the Body	518
7.5	Motion of Humans in Fluids.	519
7.5.1	Swimming	520
7.5.2	Human Flight.	523
7.6	Summary	524
	Problems	525
	References	530
8	Cardiovascular System	533
8.1	Overview of the Circulatory System and Cardiac Cycle	533
8.1.1	Circulation.	533
8.1.2	Cardiac Cycle.	541
8.1.3	Valves.	544
8.2	Physics of the Circulation System	546
8.2.1	Properties of Blood	546
8.2.2	Blood Pressure and Flow in Vessels	547
8.2.3	Capillaries and Osmotic Pressure	561
8.2.4	Blood Flow Rates and Speeds	564
8.2.5	Consequences of Clogged Arteries	572
8.2.6	Work Done by the Heart and the Metabolic Needs of the Heart	576
8.3	Strokes and Aneurysms	577
8.3.1	Hemorrhagic Strokes.	578
8.3.2	Ischemic Strokes.	587
8.4	Modeling the Circulatory System and the Heart.	588
8.4.1	Model of the Heart	588
8.4.2	Model of the Overall Flow in the Circulatory System	592
8.4.3	The Arterial Pulse.	596
8.4.4	Windkessel Model	600
8.4.5	Modeling the Malfunctioning Heart.	601
8.5	Summary	605
	Problems	605
	References	618
9	Lungs and Breathing	623
9.1	Structure of the Lungs	624
9.2	The Physics of the Alveoli	628
9.3	Physics of Breathing	632
9.4	Volume of the Lungs.	635

9.5	Breathing Under Usual and Unusual Conditions	637
9.5.1	Flow of Air During Breathing	637
9.5.2	Mechanical Model of Breathing and Model Parameters	640
9.5.3	Inspiration/Expiration Cycle	640
9.5.4	Breathing with a Diseased Lung	641
9.5.5	Breathing at Higher Elevations and Under Other Unusual Conditions	644
9.6	Work Needed to Breathe	646
9.7	Summary	647
	Problems	648
	References	654
10	Sound, Speech, and Hearing	657
10.1	The Physics of Sound Waves	658
10.1.1	The Speed and Properties of Sound Waves	659
10.1.2	Intensity of Sound Waves	661
10.1.3	What Happens When Sound Travels from One Medium to Another?	667
10.1.4	Resonant Cavities	670
10.2	Speech Production	673
10.2.1	Types of Sounds	673
10.2.2	Systems in Speech Production	677
10.2.3	Parameters of the Human Voice	691
10.2.4	The Energetics of Speaking	692
10.3	Hearing	693
10.3.1	Auditory Sensitivity	694
10.3.2	Connections to Hearing Perception	712
10.4	Other Vibrations in the Body	716
10.4.1	Cardiac and Other Sources of Sounds	716
10.5	Summary	719
	Problems	719
	References	728
11	Light, Eyes and Vision	731
11.1	Structure of the Eye.	731
11.2	Focusing and Imaging with Lenses	738
11.2.1	Image Formation.	738
11.2.2	Scientific Basis for Imaging	741
11.2.3	Combinations of Lenses or Refractive Surfaces.	745
11.3	Imaging and Detection by the Eye.	753
11.3.1	Transmission of Light in the Eye	753
11.3.2	The Eye as a Compound Lens	755
11.3.3	Accommodation	761

11.3.4	Field of View and Binocular Vision	763
11.3.5	Adjustments of Light Levels	763
11.3.6	Limitations to Visual Acuity.	765
11.3.7	Imperfect Human Vision	775
11.3.8	Correction of Vision by Eyeglasses, Contact Lenses, and Other Means	780
11.4	Types of Vision Impairment	790
11.5	Connections to Visual Perception	791
11.6	Vision in Other Animals	798
11.7	Summary	802
	Problems	802
	References	815
12	Electrical and Magnetic Properties	819
12.1	Review of Electrical Properties	820
12.2	Electrical Properties of Body Tissues	824
12.2.1	Electrical Conduction Through Blood and Tissues.	824
12.3	Nerve Conduction	825
12.3.1	Cell Membranes and Ion Distributions.	827
12.3.2	Types of Cell Membrane Excitations.	836
12.3.3	Model of Electrical Conduction Along an Axon	838
12.4	Ion Channels, Hair Cells, Balance, Taste, and Smell	849
12.5	Electrical Properties of the Heart.	851
12.6	Electrical Signals in the Brain.	859
12.7	Effects of Electric Shock	861
12.8	Magnetic Properties.	862
12.8.1	Magnetic Field from an Axon.	863
12.8.2	Magnetic Sense	864
12.9	Electromagnetic Waves	864
12.10	Summary	864
	Problems	865
	References	870
13	Feedback and Control	873
13.1	Basics of Feedback and Control	874
13.1.1	Control Theory (Advanced Topic).	876
13.2	Regulation of the Body	878
13.2.1	Regulation of Temperature.	878
13.2.2	Control of Blood Pressure	884
13.2.3	Regulation During Exercise	885
13.2.4	Regulation of the Pupil Size.	885
13.3	Summary	887
	Problems	887
	References	888

Appendix A: Symbols and Units	889
Appendix B: Locator of Major Anatomical and Anthropometric Information	895
Appendix C: Differential Equations	897
Appendix D: Similar Model Systems	905
Appendix E: Biophysics of the Human Body	911
Appendix F: Physics of Life Cycles: Pregnancy, Injuries, Diseases and Disorders, Aging, and Dying	913
Appendix G: Physics of Sports	917
Appendix H: Physics for the Body: Measurements, Imaging, and Treatment	921
Solutions to Selected Problems.	927
Index	941

Chapter 1

Terminology, the Standard Human, and Scaling

Abstract This chapter overviews the general features of the body that will be important throughout the text, and this begins with an overview of the terminology of anatomy. This is then related to how the body moves at synovial joints, and is explained in terms of the degrees of freedom of the motion of these joints. The size and features of the body and how they are related are presented for a standard human and then interrelated using scaling relationships, including allometric rules.

Several concepts will appear throughout our discussion of the human body: medical terminology, the characteristics of a “typical” human, and how body properties and responses scale with parameters. Much of the problem we have in comprehending specialists in any field is in understanding their jargon, and not in understanding their ideas. This is particularly true for medicine.

Medical terminology might seem to be intimidating, but when you examine each part of the term it becomes clearer. For example, what does “hypertrophic cardiomyopathy” mean? “Hyper” means something that is more than normal or expected. “Trophic” denotes size. “Cardio” refers to the heart. “Myo” means muscle. “Pathy” refers to disease. So hypertrophic cardiomyopathy is a disease or disorder due to the enhanced size of the heart muscle (due to thickened heart muscle). Use of acronyms as part of the medical jargon can also be a barrier. What is HCM? It is “h” ypertrophic “c” ardio “m” yopathy.

Much of medical jargon of interest to us is the terminology used in anatomy, and much of that in anatomy relates to directions and positions. To make things clearer for people who think in more physics-type terms, we will relate some of the anatomical coordinate systems used in medicine to coordinate systems that would be used by physicists to describe any physical system. We will also extend this terminology to describe the degrees of freedom of rotational motion about the joints needed for human motion. In all of our discussions we will examine a typical human. To be able to do this, we will define and characterize the concept of a standard human. The final concept in this introductory chapter will be that of scaling relationships. We will examine how the properties of a standard human scale with body mass and how the perception level of our senses varies with the level of external stimulus.

1.1 Anatomical Terminology

The first series of anatomical “coordinate systems” relate to direction, and the first set of these we encounter is right versus left. With the xyz coordinate system of the body shown in Fig. 1.1, we see that *right* means $y < 0$ and *left* means $y > 0$. Right and left, as well as all other anatomical terms, are always from the “patient’s” point of view. This was made perfectly clear to the author during a visit to his son’s ophthalmologist. When he tried to discuss what he thought was his son’s right eye, it was pointed out to the author in no uncertain terms that he was really referring to the patient’s left eye and that he was doing so in an improper manner. Case closed! (Stages in theaters have a similar convention, with stage left and stage right referring to the left and right sides of an actor on stage facing the audience. This was evident in a funny scene in the movie **Tootsie** when a stagehand was told to focus on the right side of the face of Dorothy Michaels, aka Michael Dorsey, aka Dustin Hoffman—and Dorothy heard this and then turned her (i.e., his) head so the camera would be focusing on the left side of her (i.e., his) face. A comical debate then ensued concerning whose “right” was correct, that of a person on stage or one facing the stage.)

The second direction is *superior* (or *cranial*), which means towards the head or above, i.e., to larger z . *Inferior* (or *caudal* (kaw’-dul)) means away from the head,

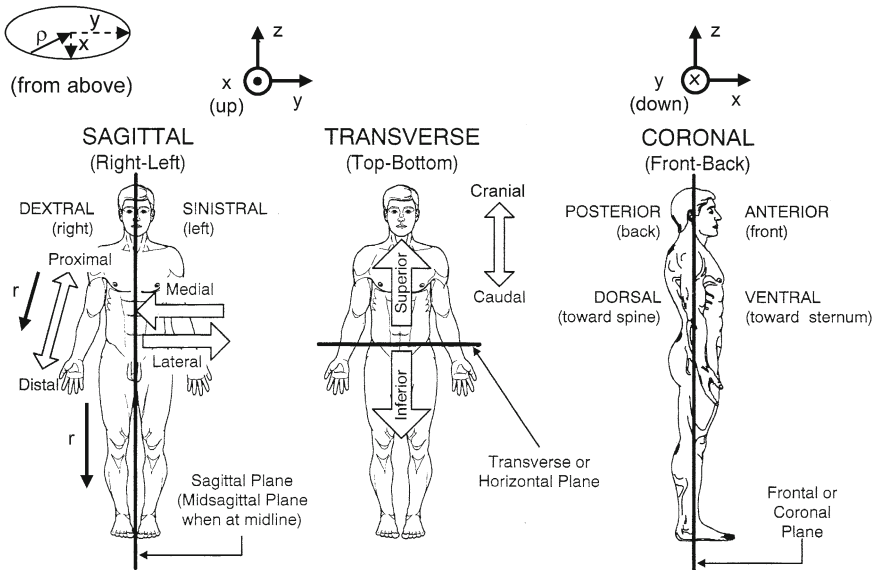


Fig. 1.1 Directions, orientations, and planes used to describe the body in anatomy, along with common coordinate systems described in the text. We will assume both terms in the following pairs mean the same: superior/cranial, inferior/caudal, anterior/ventral, and posterior/dorsal, even though there may be fine distinctions in what they mean, as is depicted here (from [16], with additions. Used with permission)

i.e., to smaller z —in an algebraic sense, so more and more inferior means smaller positive numbers and then more highly negative values of z . (This is relative to a defined $z = 0$ plane. We could choose to define the origin of the coordinate system at the center of mass of the body.) So, the head is superior to the feet, which are inferior to the head. After supplying the body with oxygen, blood returns to the heart through two major veins, the superior and inferior vena cava (vee'-nuh cave'-uh), which collect blood from above and below the heart, respectively. (As you see, words that the author has trouble pronouncing are also presented more or less phonetically, with an apostrophe after the accented syllable.)

Anterior (or *ventral*) means towards or from the front of the body, i.e., to larger x . *Posterior* (or *dorsal*) means towards or from the back, corresponding to smaller algebraic x . The nose is anterior to the ears, which are posterior to the nose.

There is another pair of terms that relate to the y coordinate, specifically to its magnitude. *Medial* means nearer the midline of the body, i.e., towards smaller $|y|$. *Lateral* means further from the midline, i.e., towards larger $|y|$.

Other anatomical terms require other types of coordinate systems. One set describes the distance from the point of attachment of any of the two arms and two legs from the trunk. Figure 1.1 depicts this with the coordinate r , where $r = 0$ at the trunk. r is never negative. *Proximal* means near the point of attachment, i.e., to smaller r . *Distal* means further from the point of attachment, or larger r .

The last series of directional terms relates to the local surface of the body. This can be depicted by the coordinate ρ (inset in Fig. 1.1), which is related to x and y in an $x - y$ plane. $\rho = 0$ on the local surface of the body. *Superficial* means towards or on the surface of the body, or to smaller ρ . *Deep* means away from the surface, or towards larger ρ .

These directional terms can refer to any locality of the body. Regional terms designate a specific region in the body (Tables 1.1 and 1.2). This is illustrated by an example we will use several times later. The region between the shoulder and elbow joints is called the brachium (brae'-kee-um). The adjective used to describe this region in anatomical terms is brachial (brae'-kee-al). The muscles in our arms that we usually call the biceps are really the brachial biceps or biceps brachii, while our triceps are really our brachial triceps or triceps brachii. The terms biceps and triceps refer to any muscles with two or three points of origin, respectively (as we will see)—and not necessarily to those in our arms.

The final set of terms describes two-dimensional planes, cuts or sections of the body. They are illustrated in Fig. 1.1. A *transverse* or *horizontal* section separates the body into superior and inferior sections. Such planes have constant z . *Sagittal* sections separate the body into right and left sections, and are planes with constant y . The *midsagittal* section is special; it occurs at the midline and is a plane with $y = 0$. The *frontal* or *coronal* section separates the body into anterior and posterior portions, as described by planes with constant x .

Much of our outright confusion concerning medical descriptions is alleviated with the knowledge of these three categories of anatomical terminology. There is actually a fourth set of anatomical terms that relates to types of motion. These are discussed in Sect. 1.2.

Table 1.1 Anatomical terms in anterior regions

Anatomical term	Common term
Abdominal	Abdomen
Antebrachial	Forearm
Axilliary	Armpit
Brachial	Upper arm
Buccal	Cheek
Carpal	Wrist
Cephalic	Head
Cervical	Neck
Coxal	Hip
Crural	Front of leg
Digital	Finger or toe
Frontal	Forehead
Inguinal	Groin
Lingual	Tongue
Mammary	Breast
Mental	Chin
Nasal	Nose
Oral	Mouth
Palmar	Palm
Pedal	Foot
Sternal	Breastbone
Tarsal	Ankle
Thoracic	Chest
Umbilical	Navel

Table 1.2 Anatomical terms in posterior regions

Anatomical term	Common term
Acromial	Top of shoulder
Femoral	Thigh
Gluteal	Buttock
Occipital	Back of head
Plantar	Sole of foot
Popliteal	Back of knee
Sacral	Between hips
Sural	Back of leg
Vertebral	Spinal column

1.2 Motion in the Human Machine

Anatomical terms refer to the body locally whether it is at rest or in motion. Since we are also concerned with how we move, we need to address human motion [3]. We will describe how we move by examining the *degrees of freedom* of our motion and the means for providing such motion by our joints. We will see that our arms and legs are constructed in a very clever manner. Because joints involve motion between bones, we will need to refer to the anatomy of the skeletal system, as in Fig. 1.2.

Think of a degree of freedom (DOF) of motion as a coordinate needed to describe that type of motion. If you want to relocate an object, you are generally interested in changing its center of mass and its angular orientation. You may want to change its center of mass from an (x, y, z) of $(0, 0, 0)$ to (a, b, c) . Because three coordinates are needed to describe this change, there are three “translational” degrees of freedom. Similarly, you can change the angular orientation of the object about the x , y , and z axes, by changing the angles this object can be rotated about these three axes: θ_x , θ_y , and θ_z , respectively. So, there are also three rotational degrees of freedom. (Sometimes, these three independent rotations are defined differently, by the three Eulerian angles, which will not be introduced here.)

These six (three plus three) degrees of freedom are independent of each other. Keeping your fingers rigid as a fist, you should be able to change independently either the x , y , z , θ_x , θ_y , and θ_z of your fist by moving your arms in different ways. You should try to change the x , y , and z of your fist, while keeping θ_x , θ_y , and θ_z fixed. Also, try changing the θ_x , θ_y , and θ_z of your fist, while keeping its x , y , and z constant.

We would like each of our arms and legs to have these six degrees of freedom. *How does the body do it?* It does it with joints, also known as *articulations*. Two types of articulations, fibrous (bones joined by connective tissue) and cartilaginous (bones joined by cartilage) joints, can bend only very little. One type of fibrous joint is the suture joint that connects the bony plates of the skull. These plates interdigitate through triangle-like tooth patterns across more compliant seams with fibers, so this joint can bear and transmit loads, absorb energy, and provide flexibility for growth, respiration, and locomotion [17, 18]. There is a joint cavity between the articulating bones in synovial joints. Only these synovial joints have the large degree of angular motion needed for motion. As seen in Fig. 1.3, in synovial joints cartilage layers on the ends of opposing bones are contained in a sac containing synovial fluid. The coefficient of friction in such joints is lower than any joints made by mankind. (More on this later.)

There are several types of synovial joints in the body, each with either one, two, or three degrees of angular motion. Each has an analog with physical objects, as seen in Fig. 1.4. For example, a common door hinge is a model of one degree of angular freedom. Universal joints, which connect each axle to a wheel in a car, have two angular degrees of freedom. A ball-and-socket joint has three independent degrees of angular motion. The water faucet in a shower is a ball-and-socket joint. The balls and sockets in these joints are spherical. Condylod or ellipsoidal joints are

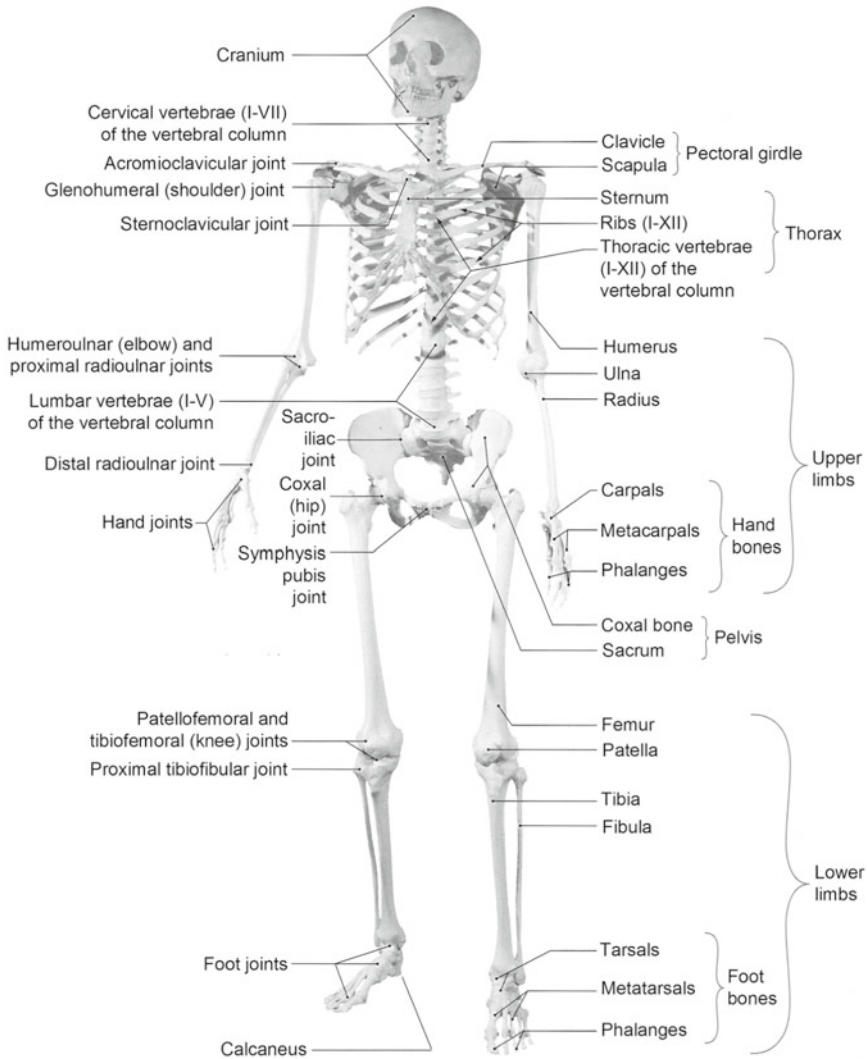


Fig. 1.2 Anatomy of the skeletal system, anterior view, with major bones and joints listed (from [37])

ball-and-socket joints with ellipsoidal balls and sockets. They have only two degrees of freedom because rotation is not possible about the axis emanating from the balls. A saddle joint, which looks like two saddles meshing into one another, also has two degrees of angular motion. Other examples are shown in Fig. 1.4.

Now back to our limbs. Consider a leg with rigid toes. The upper leg bone (femur) is connected to the hip as a ball-and-socket joint (three DOFs) (as in the song “Dry Bones” aka “Them Bones” in which “The hip bone is connected to the thigh

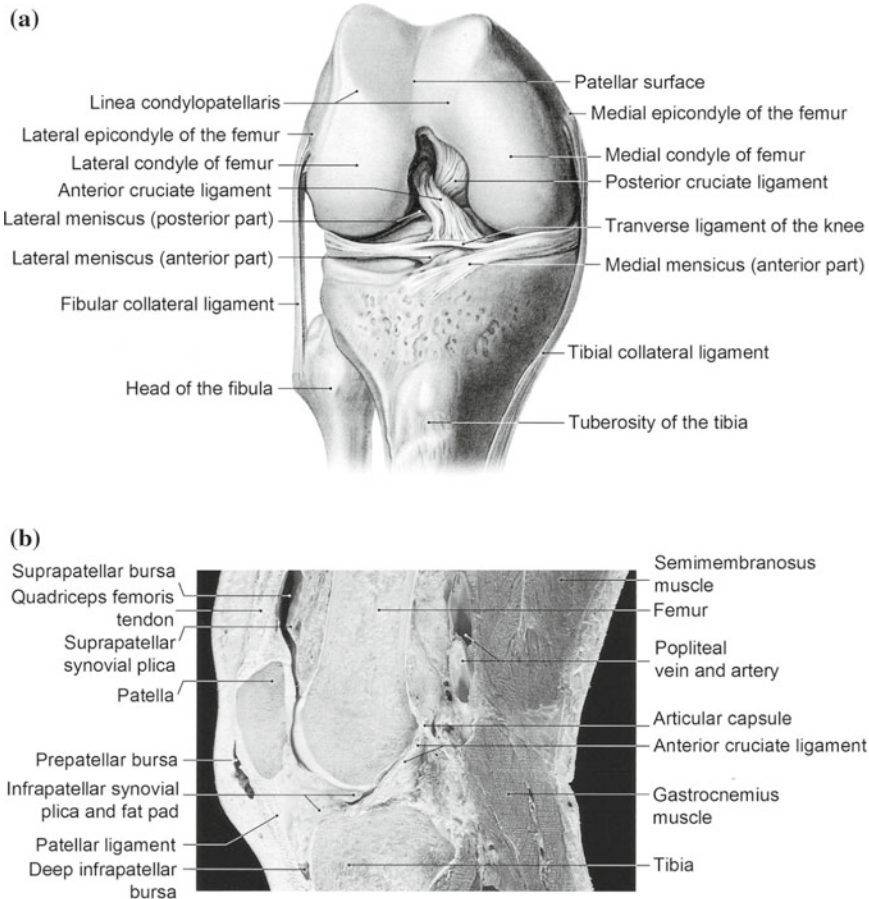


Fig. 1.3 The right knee synovial joint, with **a** anterior view with the kneecap (patella) removed and **b** in sagittal section (photo). Also see Fig. 3.3e (from [37])

bones,” The knee is a hinge (one DOF). The ankle is a saddle joint (two DOFs). This means that each leg has six degrees of angular motion, as needed for complete location of the foot. Of course, several of these degrees of freedom have only limited angular motion.

Now consider each arm, with all fingers rigid. The upper arm (humerus) fits into the shoulder as a ball-and-socket joint (three DOFs). The elbow is a hinge (one DOF). The wrist is an ellipsoidal joint (two DOFs). That makes six DOFs. The leg has these six DOFs, but the arm has one additional DOF, for a total of seven. This additional DOF is the screwdriver type motion of the radius rolling on the ulna (Figs. 1.2, 2.7, and 2.8), which is a pivot with 1 DOF. With only six DOFs you would be able to move your hand to a given x , y , z , θ_x , θ_y , θ_z position in only one way. With the additional DOF you can do it in many ways, as is seen for the person sitting in a chair in Fig. 1.5.

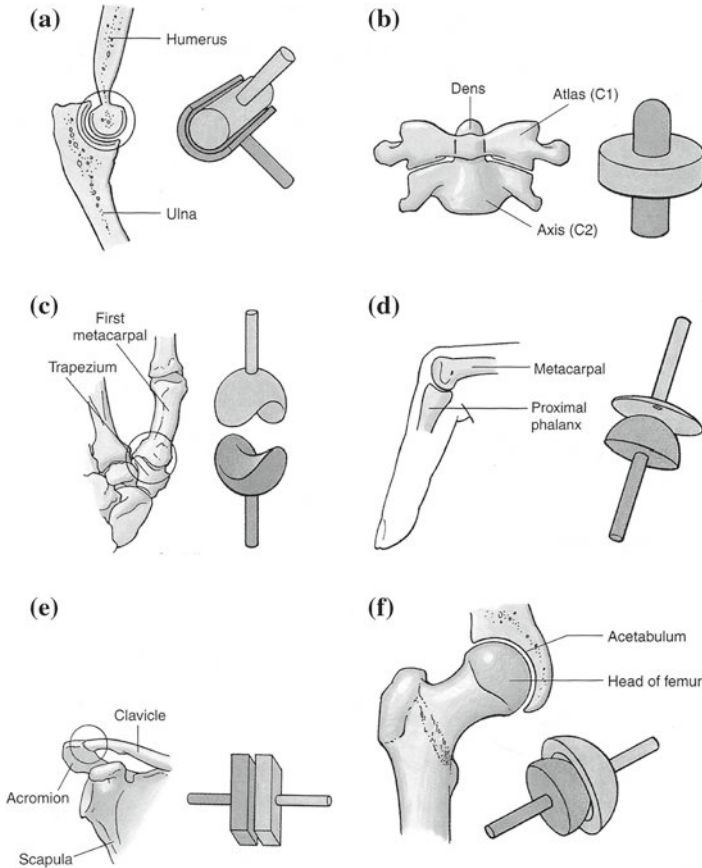


Fig. 1.4 Six types of synovial joints, including a: **a** hinge joint (1D joint), as in the elbow joint for flexion and extension, **b** pivot joint (1D joint), as in the atlantoaxial joint in the spinal cord for rotation, **c** saddle joint (2D), which is both concave and convex where the bones articulate, as in the joint between the first metacarpal and the trapezium in the hand, **d** condyloid or ellipsoidal joint (2D), as in the metacarpophalangeal (knuckle) joint between the metacarpal and proximal phalanx for flexion and extension, abduction and adduction, and circumduction, **e** plane joint (2D), as in the acromioclavicular joint in the shoulder for gliding or sliding, and **f** ball-and-socket joint (3D), as in the hip joint (and the shoulder joint) for flexion and extension, abduction and adduction, and medial and lateral rotation. See Figs. 1.9 and 1.10 for definitions of the terms describing the types of motion about joints and the diagrams in Fig. 1.11 for more information about synovial joints (from [26]. Used with permission)

There are many more degrees of freedom available in the hand, which enable the complex operations we perform, such as holding a ball. Figure 1.6 shows the bones of the hand, and the associated articulations and degrees of freedom associated with the motion of each finger.

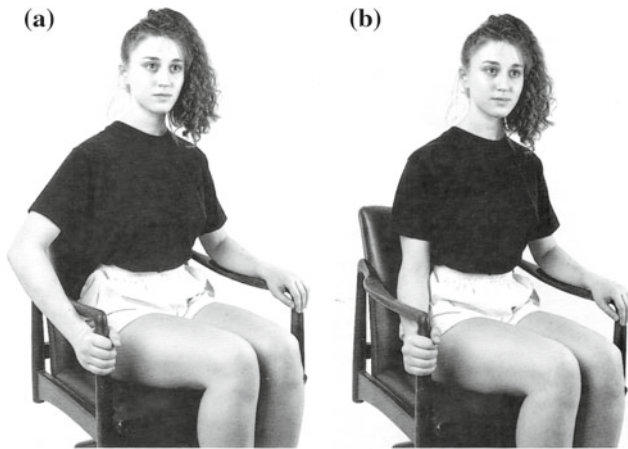


Fig. 1.5 Nonunique way of positioning the right arm. This is demonstrated by grasping the armrest while sitting, with the six coordinates of the hand (three for position and three for angle) being the same in both arm positions. This is possible because the arm can use its seven degrees of freedom to determine these six coordinates (from [3]. Copyright 1992 Columbia University Press. Reprinted with the permission of the press)

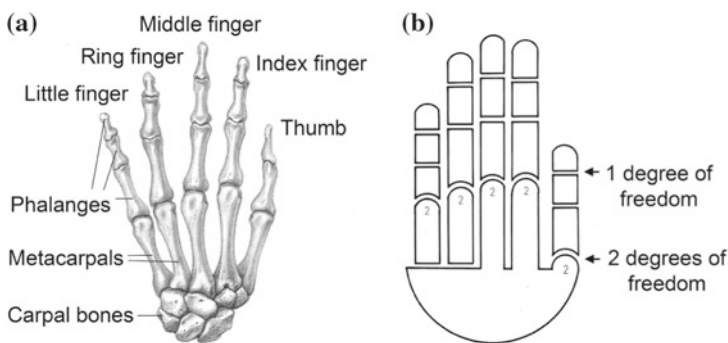


Fig. 1.6 **a** Anatomy of the hand and **b** the degrees of freedom of the hand and fingers, with joints (spaces) having one (spaces with flat terminations) or two (curved terminations, with a “2” below the joint) degrees of freedom (from [3]. Copyright 1992 Columbia University Press. Reprinted with the permission of the press)

We can also see why it is clever and good engineering that the knee hinge divides the leg into two nearly equal sections and the elbow hinge divides the arm into two nearly equal sections. In the two-dimensional world of Fig. 1.7 this enables a greater area (volume for 3D) to be covered than with unequal sections.

In preparation for our discussion of statics and motion of the body, we should consider the building blocks of human motion. There are four types of components: *bones*, *ligaments*, *muscles*, and *tendons*. Each has a very different function

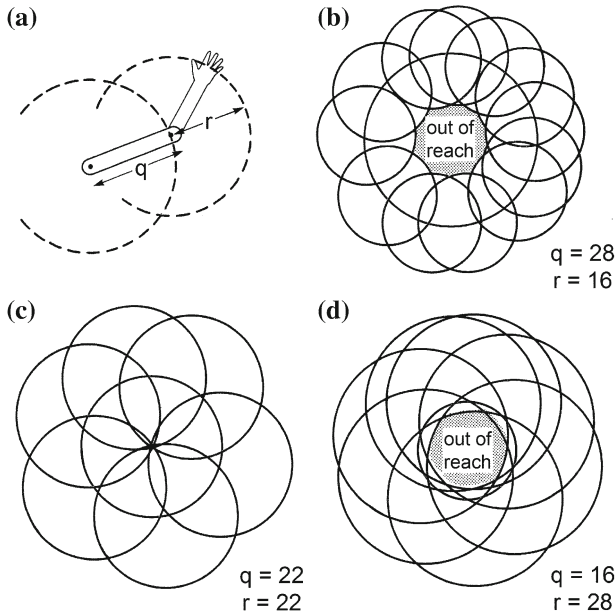


Fig. 1.7 Range of hand motion in two dimensions for different lengths of the upper and lower arms (from [3]. Copyright 1992 Columbia University Press. Reprinted with the permission of the press)

and mechanical properties. Bones are often lined with hyaline (high'u-lun) articular cartilage at the synovial joints. Ligaments hold bones together. Muscles, in particular skeletal muscles, are the motors that move the bones about the joints. (There is also cardiac muscle—the heart—and smooth muscle—of the digestive and other organs.) Tendons connect muscles to bones. Muscles are connected at *points of origin and insertion* via tendons; the points of insertion are where the “action” is. Figure 1.8 shows several of the larger muscles in the body, along with some of the tendons.

Muscles work by contraction only, i.e., only by getting shorter. Consequently, to be able to move your arms one way and then back in the opposite direction, you need pairs of muscles on the same body part for each opposing motion. Such opposing pairs, known as “antagonists,” are very common in the body.

We now return to our brief review of terminology, this time to describe the angular motion of joints. It is not surprising that these come in opposing pairs (Figs. 1.9 and 1.10) as supplied by antagonist muscles. When the angle of a 1D hinge, such as the elbow, increases it is called *extension* and when it decreases it is *flexion*. When you rotate your leg away from the midline of your body, it is *abduction*, and when you bring it closer to the midline, it is *adduction*. When you rotate a body part about its long axis it is called *rotation*. The screwdriver motion in the arm is *pronation* (a front facing hand rotates towards the body) or *supination* (away from the body), and so supination is the motion of a right hand screwing in a right-handed screw

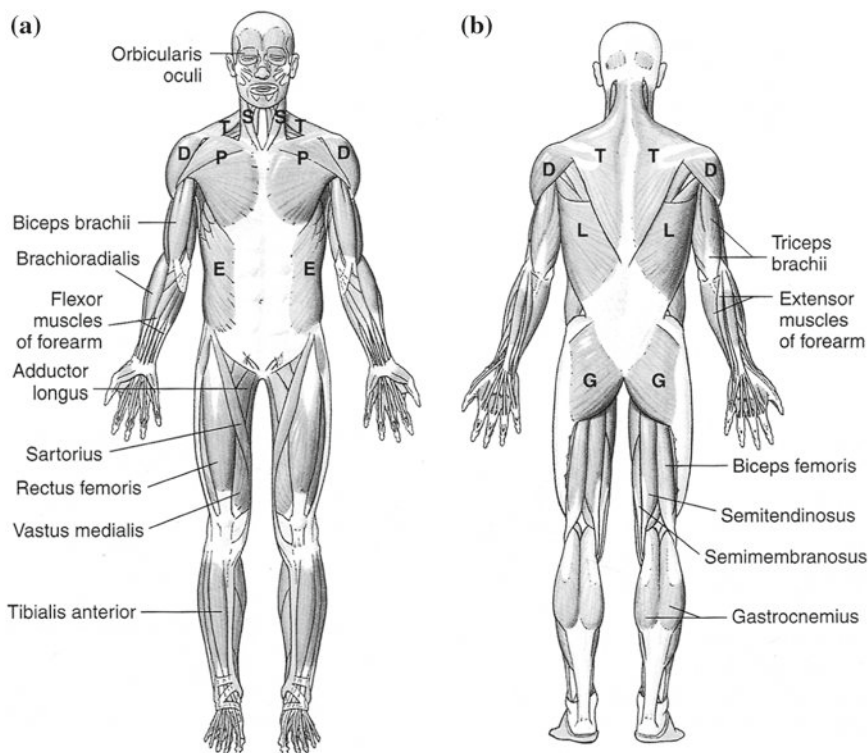


Fig. 1.8 **a** Anterior and **b** posterior views of some of the larger skeletal muscles in the body. Several muscles are labeled: *S* sternocleidomastoid, *T* trapezius, *D* deltoid, *P* pectoralis major, *E* external oblique, *L* latissimus dorsi, *G* gluteus maximus. In (**b**), the broad-banded tendon extending from the gastrocnemius and soleus (deep to the gastrocnemius, not shown) muscles to the ankle (calcaneus) is the calcaneal (or Achilles) tendon (from [26]. Used with permission)

(clockwise looking distally from the shoulder to the hand) and pronation is that of a right hand unscrewing a right-handed screw (counterclockwise looking distally from the shoulder to the hand). Examples of the rotation axes for the synovial joints used in these opposing motions are given in Fig. 1.11.

One example of opposing motion is the motion of the arm (Fig. 1.12). The biceps brachii have two points of origin and are inserted on the radius (as shown in Fig. 2.10 below). When they contract, the radius undergoes flexion about the pivot point in the elbow. The triceps brachii have three points of origin, and a point of insertion on the ulna. They are relaxed during flexion. During extension they contract, while the biceps brachii are relaxed. This is an example of a lever system about a pivot point. (This is really a pivot axis normal to the plane of the arm, as is illustrated in Fig. 1.11a for a hinge joint.)

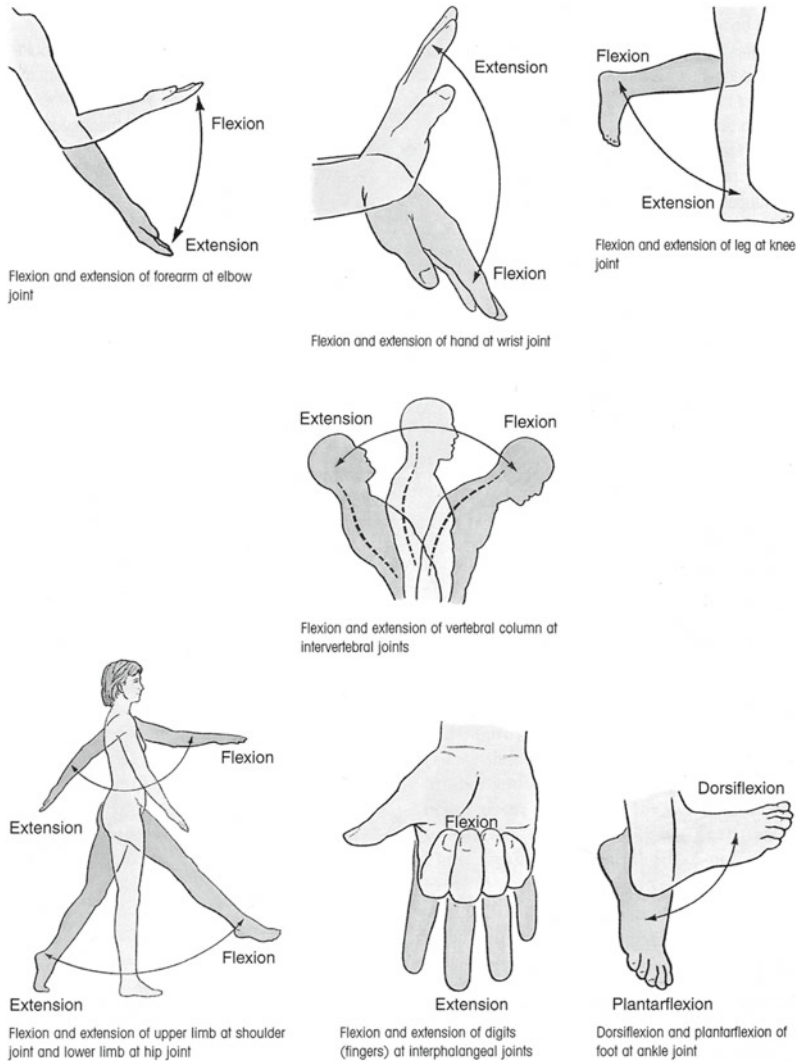


Fig. 1.9 Several antagonistic motions allowed by synovial joints. See other motions in Fig. 1.10 (from [26]. Used with permission)

A second place where there is such opposing motion is the eye. The three types of opposite motion in each eye (monocular rotations) are shown in Fig. 1.13. During *adduction* the eye turns in to the midline, while during *abduction* it turns out. The eyeball can also undergo *elevation* (eye rotating upward, or *supraduction*) or *depression* (eyeball rotating downward, or *infraduction*). Less common is the rotation of the

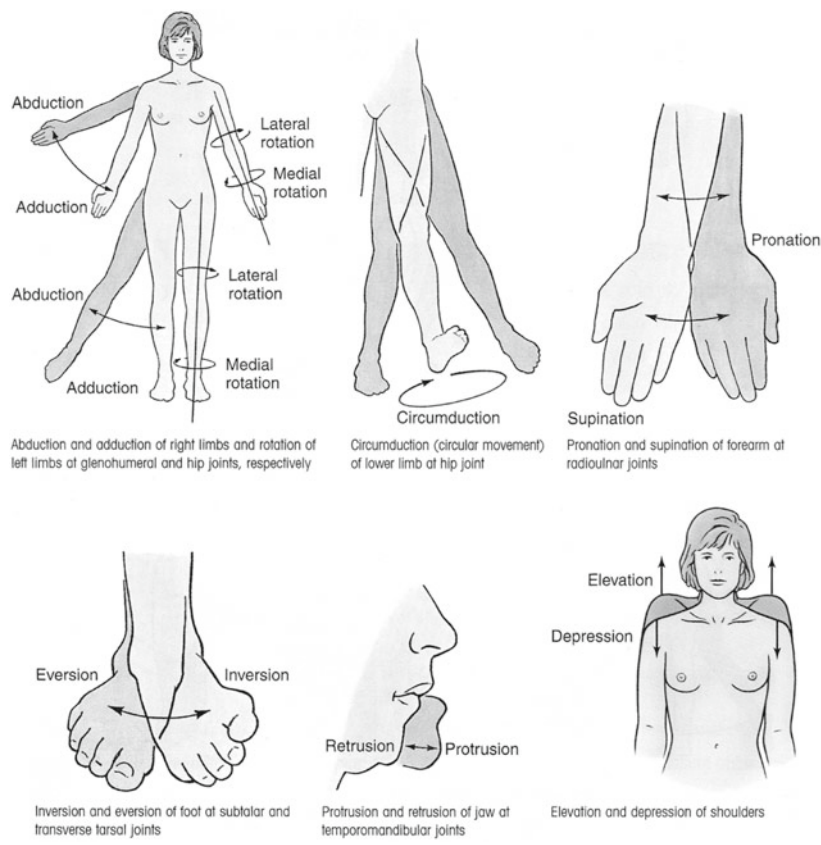


Fig. 1.10 More antagonistic motions allowed by synovial joints. See other motions in Fig. 1.9 (From [26]. Used with permission)

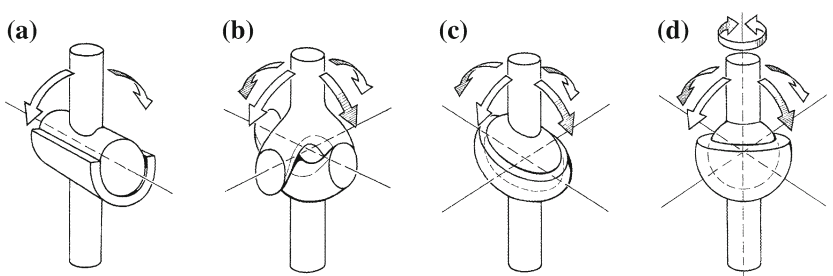


Fig. 1.11 Rotation axes for four types of synovial joints are shown for each depicted rotation direction: **a** one axis for a hinge joint (1D joint), **b** two axes for a saddle joint (2D), **c** two axes for an ellipsoidal joint (2D), and **d** three axes for a ball-and-socket joint (3D) (from [32])

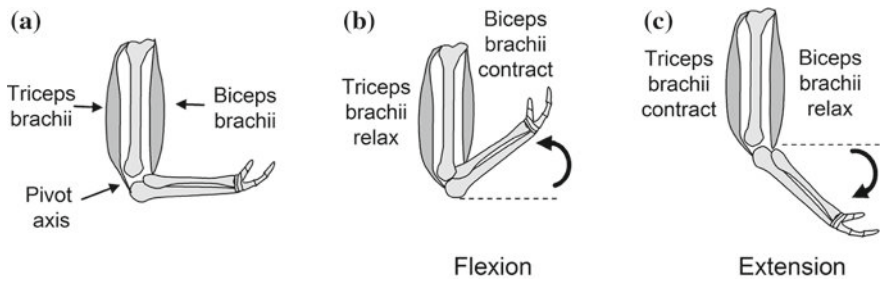


Fig. 1.12 Opposing motions of the lower arm with antagonist muscles, with flexion by contraction of the biceps brachii and extension by the contraction of the triceps brachii. The axis of rotation is seen in Fig. 1.11a

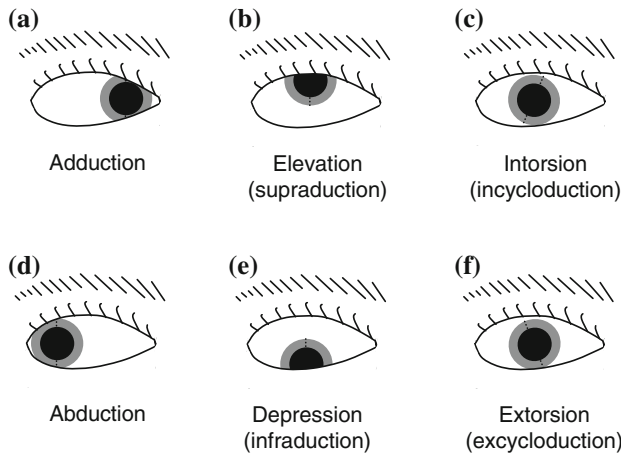


Fig. 1.13 Rotations of the right eye. A dashed line has been added across the iris to help view the rotations (based on [38])

eyeball about an axis normal to the iris, in opposing *intorsion* (*incycloduction*) or *extorsion* (*excycloduction*) motions. There are three pairs of opposing muscles per eye, each attached to the skull behind the eye, that control these motions (Fig. 1.14, Table 1.3). However, of these three pairs, only one is cleanly associated with only one of these pairs of opposing motions. Adduction occurs with the contraction of the medial rectus muscle, while abduction occurs when the lateral rectus contracts. The primary action of the superior rectus is elevation, while that of the opposing inferior rectus is depression. The primary action of the superior oblique is also depression, while that of the opposing inferior oblique is also elevation. These last two pairs of muscles have secondary actions in adduction/abduction and intorsion/extorsion that depend on the position of the eye. Binocular vision requires coordinated motion of the three opposing muscle pairs in both eyes, as described in Table 1.4.

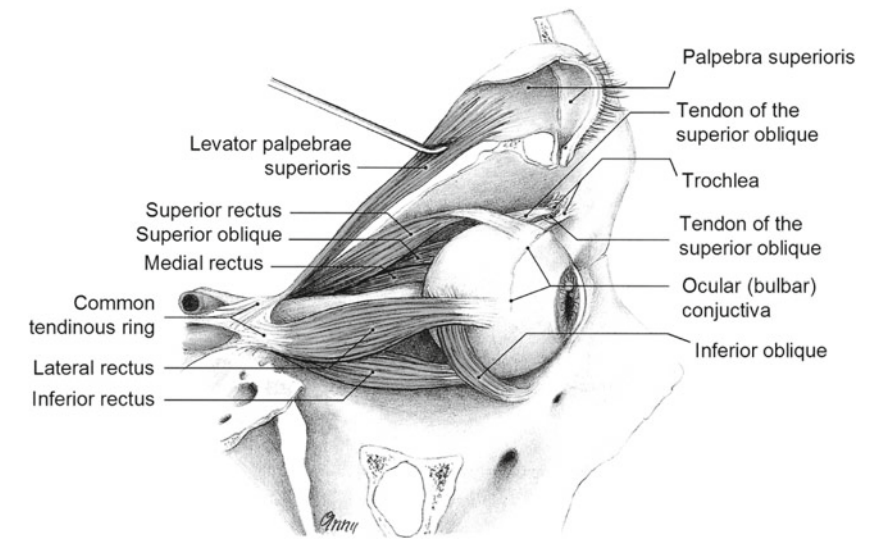


Fig. 1.14 Ocular muscles, with the eyelid (palpebra) pulled up as shown. The tendon of the superior oblique muscle (marked in two regions) passes through the trochlea loop (from [37])

Table 1.3 Ocular muscle functions

Muscle	Primary action	Secondary action
Lateral rectus	Abduction	None
Medial rectus	Adduction	None
Superior rectus	Elevation	Adduction, intorsion
Inferior rectus	Depression	Abduction, extorsion
Superior oblique	Depression	Intorsion, abduction
Inferior oblique	Elevation	Extorsion, abduction

Based on [38]

Table 1.4 Muscle combinations of both eyes for gaze directions

Direction of gaze	Right eye muscle	Left eye muscle
Eyes up, right	Superior rectus	Inferior oblique
Eyes right	Lateral rectus	Medial rectus
Eyes down, right	Inferior rectus	Superior oblique
Eyes down, left	Superior oblique	Inferior rectus
Eyes left	Medial rectus	Lateral rectus
Eyes up, left	Inferior oblique	Superior rectus

Based on [38]

1.3 The Standard Human

We will often, but not always, model humans assuming numerical values for mass, height, etc. of a “standard” human, a 70 kg man with parameters similar to those in Table 1.5. The distributions of body heights and weights, and of course their averages, differ in different regions and change with time. For example, heights of

Table 1.5 A description of the “Standard Man”

Age	30 yr
Height	1.72 m (5 ft 8 in)
Mass	70 kg
Weight	690 N (154 lb)
Surface area	1.85 m ²
Body core temperature	37.0 °C
Body skin temperature	34.0 °C
Heat capacity	0.83 kcal/kg-°C (3.5 kJ/kg-°C)
Basal metabolic rate	70 kcal/h (1,680 kcal/day, 38 kcal/m ² -h, 44 W/m ²)
Body fat	15%
Subcutaneous fat layer	5 mm
Body fluids volume	51 L
Body fluids composition	53% intracellular; 40% interstitial, lymph; 7% plasma
Heart rate	65 beats/min
Blood volume	5.2 L
Blood hematocrit	0.43
Cardiac output (at rest)	5.0 L/min
Cardiac output (in general)	3.0 + 8 × O ₂ consumption (in L/min) L/min
Systolic blood pressure	120 mmHg (16.0 kPa)
Diastolic blood pressure	80 mmHg (10.7 kPa)
Breathing rate	15/min
O ₂ consumption	0.26 L/min
CO ₂ production	0.21 L/min
Total lung capacity	6.0 L
Vital capacity	4.8 L
Tidal volume	0.5 L
Lung dead space	0.15 L
Lung mass transfer area	90 m ²
Mechanical work efficiency	0–25%

There are wide variations about these typical values for body parameters. Also, these values are different for different regions; the ones in the table typify American males in the mid-1970s. Values for women are different than for men; for example, their typical heights and weights are lower and their percentage of body fat is higher
Using data from [7, 41]

Table 1.6 Body segment lengths. Also see Fig. 1.15

Segment	Segment length ^a /body height H
Head height	0.130
Neck height	0.052
Shoulder width	0.259
Upper arm	0.186
Lower arm	0.146
Hand	0.108
Shoulder width	0.259
Chest width	0.174
Hip width/leg separation	0.191
Upper leg (thigh)	0.245
Lower leg (calf)	0.246
Ankle to bottom of foot	0.039
Foot breadth	0.055
Foot length	0.152

^aUnless otherwise specified

Using data from [42]

western European men increased by $\sim 8\text{--}17$ cm over the last two centuries, with much of these increases being very recent, with typical rates of $\sim 1\text{--}2$ cm/decade [13].

We will need details of human anatomy in some cases, and these are provided for the standard human now and in subsequent chapters as needed. We will also need to use the findings of *anthropometry*, which involves the measurement of the size, weight, and proportions of the human body. Of particular use will be anthropometric data, such as those in Table 1.6 and Fig. 1.15, which provide the lengths of different anatomical segments of the “average” body as a fraction of the body height H .

Table 1.7 gives the masses (or weights) of different anatomical parts of the body as fractions of total body mass m_b (or equivalently, total body weight W_b). The mass and volume of body segments are determined on cadaver body segments, respectively by weighing them and by measuring the volume of water displaced for segments immersed in water. (This last measurement uses Archimedes’ Principle, described in Chap. 7.) The average density of different body segments can then be determined, as in Table 1.7. The volumes of body segments of live humans can be measured by water displacement (Problem 1.39) and then their masses can be estimated quite well by using these cadaver densities. Whole body densities of live humans can be measured using underwater weighing, as is described in Problem 1.43. A relation for average body density is given below in (1.3). This is closely related to determining the percentage of body fat, as is presented below in (1.4) and (1.5).

The normalized distances of the segment center of mass from both the proximal and distal ends of a body segment are given in Table 1.8. (Problems 1.45 and 1.46 explain how to determine the center of mass of the body and its limbs.) The normalized radius of gyration of segments about the center of mass, the proximal end, and the distal end are presented in Table 1.9. (The radius of gyration provides a measure

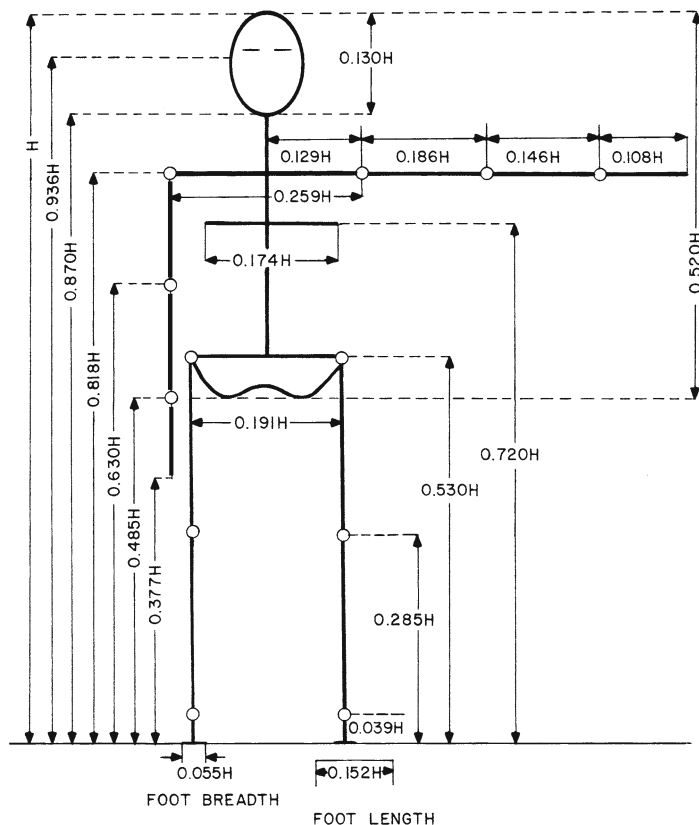


Fig. 1.15 Body segments length, relative to body height H (from [9], as from [31]. Reprinted with permission of Wiley)

of the distribution of mass about an axis, as described in (3.28) and Fig. 3.24b. Problem 3.16 describes how the radii of gyration in Table 1.9 are related.)

Note that all of the data in these different tables are not always consistent with each other because of the variations of sources and the different ranges of subjects and methods used for each table.

We have a tremendous range of mobility in our articular joints, but not as much as in the idealized joints in Fig. 1.4. The average ranges of mobility in people are given in Table 1.10 for the motions depicted in Fig. 1.16, along with the standard deviations about these values. (For normal or Gaussian distributions with an average, A , and standard deviation, SD , about 68% of all values are between $A - SD$ and $A + SD$.) Three degrees of freedom are given for the shoulder and hip, two for the wrist and the foot (listed separately as foot and ankle), and one each for the elbow and forearm. The knee, as idealized above, has one DOF, but two are listed here: the flexion in a 1 D hinge and also some rotation of the upper and lower leg about the knee.

Table 1.7 Masses and mass densities of body segments

Segment	Segment mass/total body mass m_b	Mass density (g/cm ³)
Hand	0.006	1.16
Forearm	0.016	1.13
Upper arm	0.028	1.07
Forearm and hand	0.022	1.14
Total arm	0.050	1.11
Foot	0.0145	1.10
Lower leg (calf)	0.0465	1.09
Upper leg (thigh)	0.100	1.05
Foot and lower leg	0.061	1.09
Total leg	0.161	1.06
Head and neck	0.081	1.11
Trunk	0.497	1.03

Using data from [42]

Table 1.8 Distance of the center of mass from either segment end, normalized by the segment length

Segment	Center of mass from	
	Proximal	Distal
Hand	0.506	0.494
Forearm	0.430	0.570
Upper arm	0.436	0.564
Forearm and hand	0.682	0.318
Total arm	0.530	0.470
Foot	0.50	0.50
Lower leg (calf)	0.433	0.567
Upper leg (thigh)	0.433	0.567
Foot and lower leg	0.606	0.394
Total leg	0.447	0.553
Head and neck	1.00	–
Trunk	0.50	0.50

Using data from [42]

Table 1.11 gives the mass and volumes of different systems and parts of the body. The components of a typical human cell are presented in Table 1.12. Although, most of our discussion will not concern these components of a cell, this listing is instructive because it provides a quantitative assessment of these components.

The parameters in these tables and figures are averages or typical values. There are wide variations of each body parameter due to gender and the wide range of body heights, weights, and age. There are also large variations due to differing body types. The three prototypical classifications of body types [12] illustrate this

Table 1.9 Radius of gyration of a segment, about the center of mass and either end, normalized by the segment length

Segment	Radius of gyration about		
	C of M	Proximal	Distal
Hand	0.297	0.587	0.577
Forearm	0.303	0.526	0.647
Upper arm	0.322	0.542	0.645
Forearm and hand	0.468	0.827	0.565
Total arm	0.368	0.645	0.596
Foot	0.475	0.690	0.690
Lower leg (calf)	0.302	0.528	0.643
Upper leg (thigh)	0.323	0.540	0.653
Foot and lower leg	0.416	0.735	0.572
Total leg	0.326	0.560	0.650
Head and neck	0.495	0.116	–

Using data from [42]

Table 1.10 Range of joint mobility for opposing movements, with mean and standard deviation (SD) in degrees

Opposing movements	Mean	SD
Shoulder flexion/extension	188/61	12/14
Shoulder abduction/adduction	134/48	17/9
Shoulder medial/lateral rotation	97/34	22/13
Elbow flexion	142	10
Forearm supination/pronation	113/77	22/24
Wrist flexion/extension	90/99	12/13
Wrist abduction/adduction	27/47	9/7
Hip flexion	113	13
Hip abduction/adduction	53/31	12/12
Hip medial/lateral rotation (prone)	39/34	10/10
Hip medial/lateral rotation (sitting)	31/30	9/9
Knee flexion (prone)—voluntary, arm assist	125, 144	10,9
Knee flexion—voluntary (standing), forced (kneeling)	113, 159	13, 9
Knee medial/lateral rotation (sitting)	35/43	12/12
Ankle flexion/extension	35/38	7/12
Foot inversion/eversion	24/23	9/7

The subjects were college-age males. Also see Fig. 1.16

Using data from [10], as from [4, 39]

variation: Ectomorphs have long, narrow, lean bodies with thin bones and little fat and muscle, and with hips that tend to be wider than the shoulders. Mesomorphs have solid, muscled, athletic bodies, with shoulders that tend to be broader than the

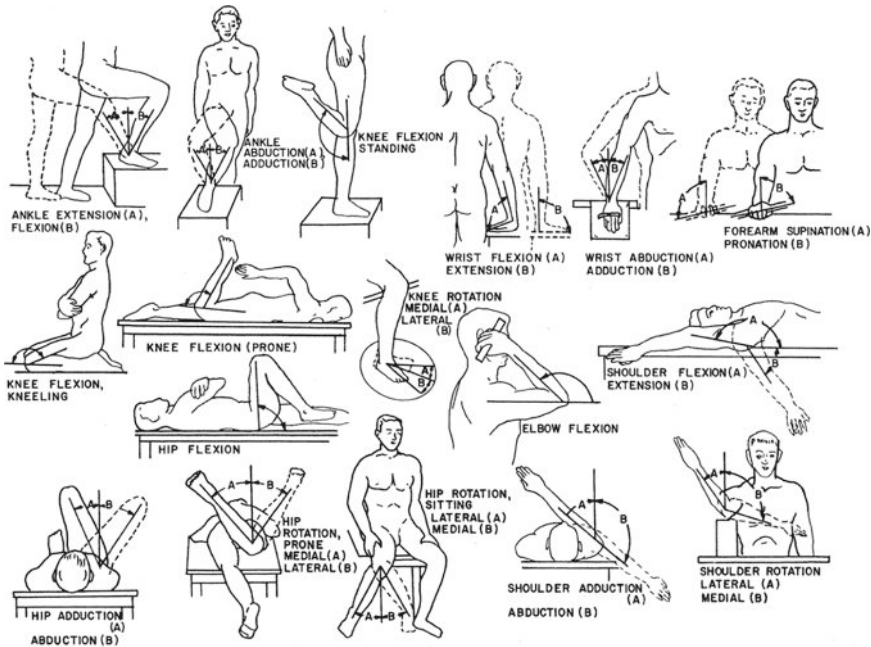


Fig. 1.16 Postures used for Table 1.10, for range of opposing motions (from [9]. Reprinted with permission of Wiley. Also see [4, 39])

hips. Endomorphs have rounded, pear-shaped bodies with large bones and excessive amounts of fatty tissue, and with shoulders that tend to be slightly wider than the hips.

More quantitative evaluation of this somatotyping can be done using the Heath-Carter technique [1]. The degree of ectomorphy is derived from how light a body is for a given height, using the body height and mass; the degree of mesomorphy is evaluated from musculoskeletal development, using the sizes of the elbow, knee, arm and calf relative to body height; and the degree of endomorphy is evaluated from fat levels, by measuring skinfolds from the arm and trunk. These can be plotted versus each other in three-dimensional plots. Ballet dancers, high jumpers and marathon runners are prototypical ectomorphs because they have large degrees of ectomorphy and small degrees of mesomorphy and endomorphy. Power athletes, such as weightlifters, boxers, and wrestlers, are mesomorphs. Sedentary middle-aged men and Sumo wrestlers are prototypical endomorphs. As the need for strength decreases and that of endurance increases, the typical body types of athletes change. Across the span from Olympic shot putters and discus throwers to sprinters to long-distance runners, the high degree of mesomorphy decreases and the low degree of ectomorphy increases.

Throughout our discussion, we will see that many processes can be described in terms of characteristic times or distances, such as the time needed for a muscle activation to decay or a molecule to diffuse in a cell. There are also more general

Table 1.11 Mass and volume of the organs of the human body

Fluid, tissue, organ, or system	Total mass (g)	Total volume (cm ³)
Adult male body	70,000	60,000
Muscle	30,000	23,000
Fat	10,500	12,000
Skin	2,000	1,800
Subcutaneous tissue	4,100	3,700
Skeleton	10,000	6,875
Gastrointestinal track	2,000	1,800
Contents (chyme/feces)	~2,000	~2,000
Blood vessels	1,800	1,700
Contents (blood)	5,600	5,400
Liver	1,650	1,470
Brain	1,400	1,350
Lungs (2)	825	775
Contents (air)	~7.7	~6,000
Heart	330	300
Chamber volume	–	450
Kidneys (2)	300	270
Urinary bladder	150	140
Contents (urine)	~500	~500
Digestive fluids	~150	~150
Pancreas	110	100
Salivary glands (6)	50	48
Synovial fluid	~50	~50
Teeth (32)	42	14
Eyes (2)	30	27
Hair (average haircut)	21	16
Gall bladder	7	7
Contents (bile)	~50	~50
Fingernails and toenails (20)	1.1	0.9

Using data from [15]

characteristic times within the human body. Your heart beats and you breathe roughly once every second. Your blood flows throughout your body roughly once every minute, and each ATP molecule (the molecule which is the ultimate form of energy usage in your body) is used and then regenerated roughly once every two minutes.

So far we have considered only the physical specifications of a typical person or a particular classification of person on Earth with the gravity and atmosphere at sea level. In subsequent chapters, we will address the physical aspects of how a typical person responds to a wide range of physical conditions, for healthy and unhealthy people, and under normal and extreme conditions. The prospect of space exploration has motivated extensive studies of how people respond to a wide range

Table 1.12 Estimated gross molecular contents of a typical 20-μm human cell

Molecule	Mass	Molecular weight (amu, daltons)	Number of molecules	Number of molecular entities
Water	65	18	1.74×10^{14}	1
Other inorganic	1.5	55	1.31×10^{12}	20
Lipid (fat)	12	700	8.4×10^{11}	50
Other organic	0.4	250	7.7×10^{10}	~200
Protein	20	50,000	1.9×10^{10}	~5,000
RNA	1.0	1×10^6	5×10^7	–
DNA	0.1	1×10^{11}	46	–

Using data from [15]

of extreme physical conditions, such as extreme pressures, temperatures, linear and rotary accelerations, collisions, vibrations, weightlessness, and sound [27].

Let us consider some of the ways our bodies change during space travel [21, 40]. With weightlessness ($g = 0$), the usual compression of intervertebral disks (see Chap. 2) on Earth does not occur, so astronauts are about an inch taller and their space suits must be made large enough so they will still fit after launch. Their heads get bigger when they leave the gravity of Earth, because blood flows to their heads faster and leaves it slower; the body partially adapts to this (Chap. 8). (Still, when considered in tandem with the lower fluid volume in the lower limbs, their appearance is sometimes referred to as the “puffy face-bird leg” syndrome.) The general increased fluid in the upper body increases intracranial pressure, which affects eyeball shape and the optic nerve, and therefore vision (Chap. 11). Exercise with free weights is not possible, so astronauts need to use exercise bands. Blood volume decreases with the need to maintain blood distribution (Chap. 8). Weightlessness affects balance through the vestibular/semicircular canal system (motion sickness) (Chap. 12). Because there is no gravity feed for food and drink down the esophagus, space travel is possible with normal eating and drinking only because of the normal peristalsis in the esophagus. Of course, astronauts need to have air for breathing (Chap. 9) and be at temperatures that sustain human life (Chap. 6). Changes in the body and their effects occur both during the early and later stages of space flight. Astronauts experience changes that sometimes have significant physical consequences after returning to Earth, such as those due to muscle (Chap. 5), skeleton (Chap. 4) and heart (Chap. 8) atrophy. There might also be consequences from inflight exposure to ionizing radiation.

1.4 Scaling Relationships

In human and animal biology there are diverse ranges of properties that scale with either a property, such as mass or length, or a physical or chemical input. We summarize them here; they relate to many concepts developed and used in subsequent chapters.

1.4.1 Allometric Rules

Some properties scale with body mass in a fairly predictable way, and are characterized by scaling relationships called *allometric rules*. For a property f for animals with body mass m_b (in kg), an allometric relation has the form

$$f(\text{in a given set of units}) = am_b^\alpha. \tag{1.1}$$

Technically, the relationship is allometric if $\alpha \neq 1$. Some examples are given in Table 1.13. By the way, *allometric* means “by a different measure” from the Greek *alloios*, which means “different”—so how body height scales with body mass is an allometric relationship. *Isometric* means “by the same measure”—so how leg mass scales with body mass is an example of an isometric relationship. For a delightful discussion on allometry and scaling see [23]. Other equally intriguing discussions have been presented in [2, 6, 7, 22, 24, 30, 33, 34].

These relationships can hold for many species of a given type, such as land-based mammals, etc. Some are also valid within a species, such as for man—and as such would be called anthropometric relationships. Sometimes they apply only to adults in a species, and not across all age groups. See Problems 1.57 and 1.58 for more on this.

The “predicted” values from a scaling relation would be the expected average values only if the parameters for all species follow the relation *exactly*—and there is no reason why this must be so. Moreover, there is always a spread, or dispersion,

Table 1.13 Allometric parameters (1.1) for mammals

Parameter	a	α
Basal metabolic rate (BMR), in W	4.1	0.75
Body surface area, in m ²	0.11	0.65
Brain mass in man, in kg	0.085	0.66
Brain mass in nonprimates, in kg	0.01	0.7
Breathing rate, in Hz	0.892	−0.26
Energy cost of running, in J/m·kg	7	−0.33
Energy cost of swimming, in J/m·kg	0.6	−0.33
Effective lung volume, in m ²	5.67×10^{-5}	1.03
Heart beat rate, in Hz	4.02	−0.25
Heart mass, in kg	5.8×10^{-3}	0.97
Lifetime, in y	11.89	0.20
Muscle mass, in kg	0.45	1.0
Skeletal mass (terrestrial), in kg	0.068	1.08
Speed of flying, in m/s	15	0.167
Speed of walking, in m/s	0.5	0.167

Using data from [2, 33]

about these average values. Some of these allometric relationships are empirical, and others can be derived, or at least rationalized, as we will see for Kleiber's Law of basal (6.19) (i.e., minimum) metabolic rates (BMRs) in Chap. 6.

One obvious example of such scaling is that the legs of bigger mammals tend to be wider in proportion to their overall linear dimension L (and mass m_b) than those for smaller mammals, and this is reflected in the larger ratio of the width of their long skeletal bones, w_{bone} , to the length, L_{bone} , for larger m_b . Such bones need to support their body weight or mass, which are proportional to their volume $\sim L^3$ because the mass densities are very nearly the same for all mammals. Such long bones in the different mammals can withstand approximately the same force per unit cross-sectional area, which means that they have the same ultimate compressive stress or UCS. (The UCS is a measure of fracture conditions that will be discussed more in Chap. 4.) If this maximum force is a fixed multiple of the weight, so it is $\propto L^3$, and the cross-sectional area is $\sim w_{\text{bone}}^2$, then $w^2 \propto L^3$ or $w \propto L^{3/2}$. Because the bone length $L_{\text{bone}} \propto L$, we see that $w/L_{\text{bone}} \propto L^{1/2} \propto m_b^{1/6}$. Body mass m_b increases by 10^4 from rat to elephant, and so this ratio increases by $10^{2/3} \sim 4$. This means that the body shapes and the bones themselves are not geometrically similar.

In some cases more accurate scaling relations necessitate the use of parameters in addition to the body mass. One example is the scaling of the surface area of a person A (in m^2), which is empirically seen to depend on height H (in m), as well as body mass m_b (in kg):

$$A = 0.202 m_b^{0.425} H^{0.725}. \quad (1.2)$$

Accurate scaling relationships often involve the index W_b/H^p (or equivalently m_b/H^p) with p ranging from 1.0 to 3.0. (Sometimes the reciprocal or square root or cube root of this parameter is used. Complicating matters further, different sets of units are commonly used for these parameters.) One such index is the specific stature or ponderal index $S = H/m_b^{1/3}$ (for which $p = 3$), which is used in the Harris–Benedict versions of Kleiber's Law specialized for people (6.30) and (6.31). It is also used in the expression for the average human density

$$\rho \text{ (in kg/L or g/cm}^3\text{)} = 0.69 + 0.9S \quad (1.3)$$

where within S the units of H are m and those of m_b are kg. Another scaling parameter is the body mass index (BMI), or Quételet's index (Q), $\text{BMI} = m_b/H^2$ (for which $p = 2.0$). It is often considered the best index for epidemiological studies. For example, with m_b in kg and H in m, the average fat content of the body increases with this index:

$$\text{Men: fat (\% of body weight)} = 1.28 \text{ BMI} - 10.1 \quad (1.4)$$

$$\text{Women: fat (\% of body weight)} = 1.48 \text{ BMI} - 7.0. \quad (1.5)$$

Table 1.14 Size-independent dimensionless groups in mammals

Parameter	a	α
Breathing flow rate/blood flow rate ^a	2.0	0.00
Mass of blood/mass of heart	8.3	0.01
Time for 50% of growth/lifespan in captivity	0.03	0.05
Gestation period/lifespan in captivity	0.015	0.05
Breathing cycle/lifespan in captivity	3×10^{-9}	0.06
Cardiac cycle/lifespan in captivity	6.8×10^{-10}	0.05
Half-life of drug ^b /lifespan in captivity	0.95×10^{-5}	0.01

The value of a in (1.1) is that for a 1 kg mammal. Also see Chaps. 8 and 9

^a(Tidal volume/breath time)/(heart stroke volume/pulse time)

^bMethotrexate

Using data from [23]

Normal or ideal body fat is 14–20% in men and 21–27% in women. A person is considered overweight when the BMI is >20 and obese when it is >30 [1]. Mortality increases as the BMI increases or decreases much from normal ranges.

Some properties do not scale with mass. From mouse to elephant, a mass range of over 10,000, the maximum jumping height (of the center of mass) of most every mammal is within a factor of 2 of $2/3$ m. Similarly, the maximum running speed of most mammals is within a factor of 2 of 7 m/s (15 mph). The main reason behind these two relationships is that the force a muscle can exert is proportional to its cross-sectional area and therefore it varies as the square of the characteristic linear dimension, such as height H . This is explored in Problems 3.43, 3.44, and 3.58 in Chap. 3. The dimensionless ratios of some physical and physiological properties are noteworthy in that they are essentially independent of size (and mass), as is seen in Table 1.14.

1.4.2 Scaling in the Senses

A very different type of scaling is exhibited by Stevens’ Law, which characterizes how the perceived strength P of a sense varies with the intensity of a stimulus S for a given sensation. This scaling is

$$P = K(S - S_0)^n \tag{1.6}$$

above a threshold S_0 [35, 36]. As seen in Table 1.15, sometimes the perception of a sense, called the *psycho perception*, is sublinear with the strength of the stimulus and sometime it is superlinear. Three of the senses involve very significant physical aspects: hearing (Chap. 10, loudness) and seeing (Chap. 11), which we will examine in detail, and touch (Chap. 2, vibration, temperature, pressure on palm, heaviness; Chap. 12, electric shock), which we will study in less detail. These three also have

Table 1.15 Exponent n for perceived strength (P) of a stimulus (S) above a threshold S_0 , with $P = K(S - S_0)^n$ in Steven’s law

Psychoperception	n	Stimulus
Brightness	0.33, 0.5	5° target, point source—dark adapted eye
Loudness	0.54, 0.60	Monoaural, binaural
Smell	0.55, 0.60	Coffee odor, heptane
Vibration	0.6, 0.95	250, 60 Hz—on finger
Taste	0.8, 1.3, 1.3	Saccharine, sucrose, salt
Temperature	1.0, 1.6	Cold, warm—on arm
Pressure on palm	1.1	Static force on skin
Heaviness	1.45	Lifted weights
Electric shock	3.5	60 Hz through fingers

Using data from [35, 36]

very important chemical and biological origins. The senses of taste and smell are essentially solely chemical and biological in basis and will not be discussed any further, except for a brief discussion of the electrical properties of the membranes of the taste and smell sensory neurons in Chap. 12. For these three other senses, we will concentrate mostly on the physical input and the beginning of the sensation process (detection). The nonlinearities inherent in Stevens’ Law are due in part to the detection process. The final parts of detection process are the generation and transmission of neural signals sent to the brain—which we will cover—and the processing in the brain—which we will cover only briefly. There are approximately 12 orders of magnitude sensitivity in hearing and vision.

1.5 Summary

Much about the body can be understood by learning the terminology of directions and local regions in the body. Much about the motion of the body can be explained by examining the rotation of bones about joints. In analyzing the physics of the body, reference can be made to anthropometric data on body parts for a standard human. Many phenomena concerning anatomy and physiology can be characterized and understood by using scaling relations.

Problems

Body Terminology

- 1.1 (a) Is the heart superior or inferior to the large intestine?
- (b) Is the large intestine superior or inferior to the heart?

- 1.2** (a) Is the navel posterior or anterior to the spine?
(b) Is the spine posterior or anterior to the navel?
- 1.3** Is the nose lateral or medial to the ears?
- 1.4** Are the eyes lateral or medial to the nose?
- 1.5** Is the foot proximal or distal to the knee?
- 1.6** Is the elbow proximal or distal to the wrist?
- 1.7** Is the skeleton superficial or deep to the skin?
- 1.8** The blind spot in the eye retina is said to be nasal to the fovea (center of the retina). What does this mean?
- 1.9** What would you expect the term *cephalid* to mean? What would be an equivalent term?
- 1.10** Which is the anterior part of the heart in Fig. 8.7? Is this a superior or inferior view of the heart?
- 1.11** Consider the directional terms *ipsilateral* and *contralateral*. One means on the same side of the body, while the other means on opposite sides of the body. Which is which?
- 1.12** The directional term *intermediate* means “in between.” What is intermediate between the upper and lower legs?
- 1.13** Is the lower arm more supinated when throwing a baseball or football, and why is this so?
- 1.14** Encephalitis is the inflammation, i.e., “itis”, of what?
- 1.15** *Presbyopia* refers to disorders in vision due to old age, such as lack of accommodation in the crystalline lens (see Chap. 11). *Presbycusis* refers to old age-related auditory impairments (see Chap. 10). What parts of these two terms mean old age, vision, and hearing?
- 1.16** What is the difference between *presbyphonia* and *dysphonia*?
- 1.17** The three tiny bones in the middle ear, the malleus, incus, and stapes are interconnected by the incudomalleolar articulation and the incudostapedial joint. Describe the origin of the names of these connections.
- 1.18** The quadriceps muscles in the upper leg attach to the kneecap (patella) through the quadriceps tendon. The kneecap is connected to the tibia by connective tissue that is sometimes called the patellar tendon and sometimes the patellar ligament. Explain why both designations have merit and why neither designation completely describes the linkage perfectly well by itself.

1.19 Consider the drawing of the hand skeleton and the schematic of a hand showing joints with one or (labeled 2) two degrees of freedom in Fig. 1.6.

- (a) How many degrees of freedom does each hand have? (Ignore the wrist joint.)
- (b) Do we need so many degrees of freedom? Why? (There is no right or wrong answer to this part. Just think about what a human hand should be able to do (in clutching, etc.) and try to express your conclusions in terms of degrees of freedom.)

1.20 Estimate the angle of each of the joints in the hand for each of the following functions [29]. (Define the angle of each joint as shown in the left hand in Fig. 1.6b to be 0° . Define rotations into the paper and clockwise motions in the plane of the paper as being positive.)

- (a) lifting a pail (a hook grip)
- (b) holding a cigarette (a scissors grip)
- (c) lifting a coaster (a five-jaw chuck)
- (d) holding a pencil (a three-jaw chuck)
- (e) threading a needle (a two-jaw pad-to-pad chuck)
- (f) turning a key (a two-jaw pad-to-side chuck)
- (g) holding a hammer (a squeeze grip)
- (h) opening a jar (a disc grip)
- (i) holding a ball (a spherical grip)

1.21 We said that the seven DOFs available for arm motion enabled nonunique positioning of the hand, but analogous nonunique positioning of the foot is not possible because the leg has only six DOFs. Use Table 1.10 to explain why this is not exactly correct.

1.22 Use Table 1.3 to show that the coordinated eye motions in Table 1.4 use the muscles listed for primary motion.

1.23 Consider a limb, of length L , composed of upper and lower limbs with respective lengths r_1 and r_2 , with $L = r_1 + r_2$. There is a total range of motion in the angles the upper limb makes with the torso and the lower limb makes with the upper limb. Assume motion only in two dimensions (see Fig. 1.7).

- (a) What area is subtended by the end of the lower limb (hand or foot) when $r_1 = r_2$?
- (b) What area is subtended by the end of the lower limb when $r_1 > r_2$? What fraction of that in (a) is this?
- (c) What area is subtended by the end of the lower limb when $r_1 < r_2$? What fraction of that in (a) is this?

1.24 Redo Problem 1.23 in three dimensions, finding the volume subtended by the end of the lower limb in each case.

The Standard Human

1.25 Qualitatively explain the differences of density in Table 1.7 in the different segments of the body. The average densities of blood, bone, muscle, fat, and air (in the lungs) can be determined from Table 1.11.

Table 1.16 An alternative set of relations of weights of body segments (all in lb)

Segment	Segment weight
Head	$0.028W_b + 6.354$
Trunk	$0.552W_b - 6.417$
Upper arms	$0.059W_b + 0.862$
Forearms	$0.026W_b + 0.85$
Hands	$0.009W_b + 0.53$
Upper legs	$0.239W_b - 4.844$
Lower legs	$0.067W_b + 2.846$
Feet	$0.016W_b + 1.826$

Using data from [25], which used unpublished data by [11]

1.26 (a) Use Table 1.7 to determine the average density of the body.

(b) Use this to determine the average volume of a 70 kg body.

Your answers will be a bit different from the rough volume estimate given in Table 1.11.

1.27 (a) Calculate the range of segment masses alternatively using Tables 1.7 and 1.16, for each type of segment listed in the latter table, for people with masses in the range 40–100 kg.

(b) Give several reasons why these ranges seem to be different.

1.28 (a) Show that (1.3) becomes

$$\rho(\text{in kg/L or g/cm}^3) = 0.69 + 0.0297S, \quad (1.7)$$

when $S = H/W_b^{1/3}$ is expressed with H in inches and W_b in lb.

(b) Show that the average density for an adult of height 5 ft 10 in and weight 170 lb is 1.065 g/cm^3 , with $S = 12.64$.

(c) Show that the average density for an adult of height 1.78 m and mass 77.3 kg is 1.066 g/cm^3 , with $S = 0.418$.

1.29 What percentage of body mass is fat, skin, the skeleton, blood, liver, the brain, the lungs, heart, kidneys, and eyes?

1.30 Use Table 1.11 to determine the mass density of blood, skin, the lungs, the air in the lungs, fat, liver, hair, eyes, and blood vessels.

1.31 In modeling heat loss in Chap. 6, a typical man is modeled as a cylinder that is 1.65 m high with a 0.234 m diameter. If the human density is 1.1 g/cm^3 , what is the mass (in kg) and weight (in N and lb) of this man?

1.32 (a) If a man has a mass of 70 kg and an average density of 1.1 g/cm^3 , find the man's volume.

(b) If this man is modeled as a sphere, find his radius and diameter.

- (c) If this man is 1.72 m high, and is modeled as a right circular cylinder, find the radius and diameter of this cylinder.
- (d) Now model a man of this height and mass as a rectangular solid with square cross-section, and find the length of the square.
- (e) Repeat this for a constant rectangular cross-section, and determine the sizes if the long and short rectangle dimensions have a ratio of either 2:1, 3:1, or 4:1.
- (f) In each above case calculate his surface area and compare it to that predicted by (1.2) for a 1.72-m tall man. Which of the above models seems best?

1.33 The cylindrical model of a man in Fig. 1.17 was once used in studies of convective cooling. What are the volume, mass, and exposed surface area (including the bottom of the lower limb) for this person? Assume each finger is 3.5 in long and has a diameter of 0.875 in and that the mass density of all components is 1.05 g/cm^3 .

1.34 How much heavier is someone with a totally full stomach, small intestine, large intestine and rectum, than when each system is empty? Assume the mass density of the contents is 1 g/cm^3 . Express your answer in mass (kg) and weight (N and lb). (Use Table 7.4.)

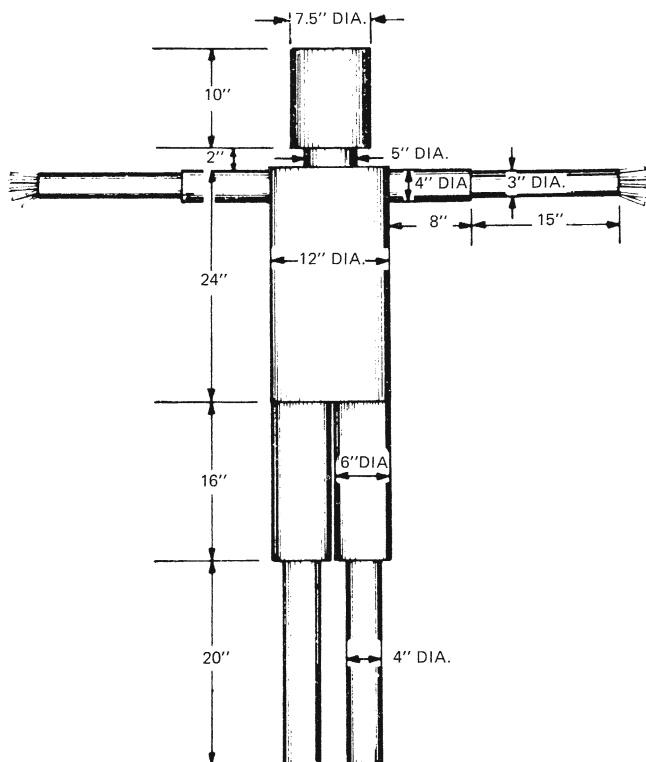


Fig. 1.17 Cylindrical model of a man used in studies of convective cooling (from [5], adapted from [28])

1.35 Use the anthropometric data to determine the average cross-sectional area and diameter of an arm and a leg of a 70 kg man. Assume the cross section of each is circular.

1.36 Compare the surface area of the standard man given in Table 1.5, alternatively as predicted by (1.1) and (1.2). Use the data given in Table 1.13.

1.37 Compare the surface area of a 50 kg, 5 ft 5 in woman, alternatively as predicted by (1.1) and (1.2). Use the data given in Table 1.13.

1.38 For an adult, the average fractional surface area is 9% for the head, 9% for each upper limb, 18% both for front and back of the torso, and 18% for each lower limb. (The remaining 1% is for genitalia.)

(a) This is used to estimate the fraction of damaged area in burn victims. It is known as the “Rule of Nines.” Why?

(b) Use the data given in Table 1.13 to determine the average surface area for each of these parts of the body for the standard man.

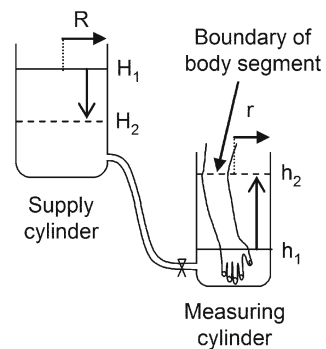
1.39 In the system depicted in Fig. 1.18, a body segment is put in the measuring cylinder and the valve is opened to allow flow of water up to the “beginning” of the body segment (giving the “1” heights). The valve is then opened until water flows into the measuring cylinder to the “end” of the body segment (giving the “2” heights). Explain how this can be used to measure the volume of the body segment.

1.40 (a) Calculate the BMI (also known as Quételet’s index) and the specific stature (also known as the ponderal index) of a person of average density ρ modeled as a cube of length L .

(b) How do these change if the person has the same overall mass, but is modeled as a rectangular solid of height H , width $0.20H$, and depth $0.15H$?

1.41 (a) A person with mass M is modeled as a rectangular solid of height H , width $0.25H$, and depth $0.25H$. The person loses weight, maintaining the same mass density, and then has a width $0.20H$ and depth $0.15H$. Calculate the BMI and the specific stature of the person before and after the weight loss.

Fig. 1.18 Immersion technique for measuring the volume of various body segments, with the *solid lines* denoting the initial water level and the *dashed lines* the final water level (based on [25]). For Problem 1.39



(b) Would you expect the mass density of the person to change during the weight loss? If so, how would you expect it to change?

1.42 (a) Explain why the use of the BMI as a metric of ideal weight would be problematic if body mass and volume scaled as the cube of the body height, because it depends inversely as the square of body height [13, 19].

(b) Specifically, if the BMI of a person who is 1.75 m tall is 25.0 kg/m^2 , find the (usual) BMI for the same person who is either 1.68 or 1.83 m tall, assuming their body mass and volume scale as the cube of body height.

(c) The ideal BMI seems to decrease with body height. Would revising the BMI index so it varies as a power of m_b/H^3 (such as $1/S^3$, where S is the specific stature) make the ideal BMI less sensitive to height?

1.43 You can determine the density and percentage of fat in people by weighing them underwater, as in Fig. 1.19. Data for two men with the same height and mass, but with different underwater masses are given in Table 1.17.

(a) Why are the volumes given as listed?

(b) What assumption has been made about the relative densities of fat and the average of the rest of the body?

(c) Is the value assumed for the density for the rest of the body reasonable? Why or why not?

(d) Table 1.17 uses the Siri formula for the percentage of body fat: $100(4.95/(\text{body density}) - 4.50)$. How do the results for the percentage and mass of body fat differ using the Brozek formula: $100(4.57/(\text{body density}) - 4.142)$ for the two cases in the table? (Both formulas may, in fact, give values for body fat percentage that are too high.)

1.44 Explain why in Problem 1.43 you can either measure the weight of the water displaced by the body or the weight of the body when it is completely submerged [14].

Fig. 1.19 Determining human body fat content and density by weighing a person in water (photo by Clifton Boutelle, News and Information Service, Bowling Green State University. Used with permission of Brad Phalin. Also see [14]). For Problem 1.43

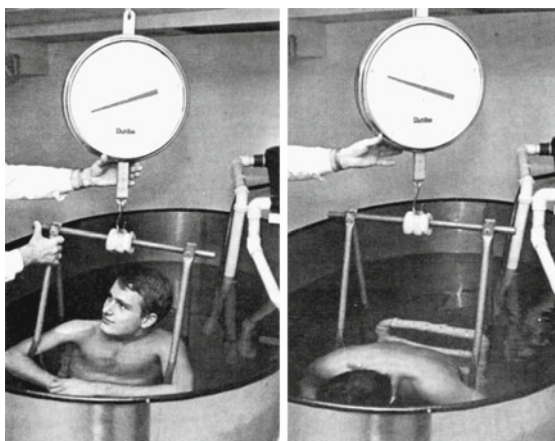


Table 1.17 Comparison of the density and fat percentage for two men with the same height and mass, but different underwater masses

Parameter	Man A	Man B
Height, m (in)	1.88(74)	1.88(74)
Mass, kg (lb)	93(205)	93(205)
Underwater mass, kg	5.00	3.50
Volume, L	88.0	89.5
Volume _{corrected} ^a , L	86.5	88.0
Body density ^b , g/cm ³	1.075	1.057
Relative fat, %	10.4	18.4
Fat mass, kg (lb)	9.7(21.4)	17.1(37.7)
Fat-free mass, kg (lb)	83.3(183.6)	75.9(167.3)

^aThe volume is corrected for the water density, intestinal gas volume, and residual lung volume

^bThe body density is the mass-corrected volume. Relative fat (in %) = $100(4.95/(\text{body density}) - 4.50)$

Using data from [41]. For Problem 1.43

1.45 You can measure the location of the anatomical center of mass of the body using the arrangement in Fig. 1.20a. The weight (w_1) and location of the mass (x_1) of the balance board are known along with the body weight w_2 . The location of the body center of mass relative to the pivot point is x_2 . The distance from the pivot to the scale is x_3 . With the body center of mass to the left of the pivot point there is a measurable force S on the scale (under the head). Show that

$$x_2 = \frac{Sx_3 - w_1x_1}{w_2}. \quad (1.8)$$

1.46 You can determine the weight of the lower part of a limb (w_4) using the same balance board as in Problem 1.45, using Fig. 1.20b. The center of mass of the limb changes from x_4 to x_5 relative to the pivot point when the limb is set vertically; concomitantly the scale reading changes from S to S' . Show that

$$w_4 = \frac{(S' - S)x_3}{(x_4 - x_5)}. \quad (1.9)$$

The location of the center of mass of the limb relative the joint near the trunk is assumed to be known. To determine the weight of the entire limb the subject should be lying on his or her back and the entire limb is flexed to a right angle.

- 1.47** (a) Determine the goal mass (in kg, and find the weight in lb) to achieve 10% fat for the two men described in Table 1.17, by using the fat-free mass.
 (b) How much fat mass (and weight) must be lost by each to attain this goal?

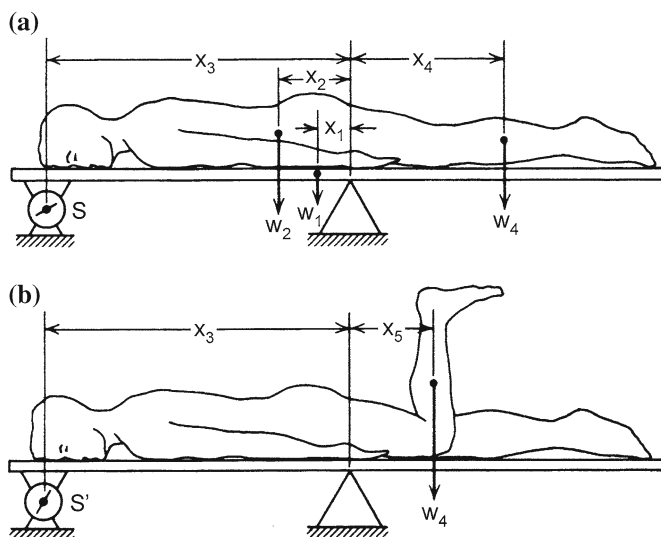


Fig. 1.20 In vivo estimation of **a** body center of mass and **b** mass of a distal segment, for Problems 1.45 and 1.46 (from [42]. Reprinted with permission of John Wiley & Sons)

1.48 The normalized distances of the segment center of mass from the proximal and distal ends in Table 1.8 always sum to 1. Is this a coincidence, a trivial point, or significant? Why?

4 Allometry and Scaling

1.49 Determine the parameters for a 70 kg person for each set of allometric relation parameters in Table 1.13. How do they compare with similar parameters listed in Tables 1.5 and 1.11?

1.50 Derive the allometric laws for the percentages of the total body mass residing in the brain, heart, muscle, and skeletal mass for mammals (such as humans).

1.51 Compare the prediction of the fat in a standard man using the BMI, with those listed in Table 1.11.

1.52 Compare the % body fat in:

- (a) a male and female who are both 5 ft 6 in, 140 lb.
- (b) males who are 6 ft 2 in and 5 ft 8 in tall, both weighing 190 lb.

1.53 For a 70 kg person living 70 years, determine the person's total lifetime

- (a) number of heart beats
- (b) number of breaths
- (c) energy consumed
- (d) energy consumed per unit mass.

1.54 Does it make sense that the ratio of the volumetric flow rates in the respiratory and circulatory systems in mammals (first entry line in Table 1.14) is essentially independent of mammal mass? Why?

1.55 Use Table 1.14 to find the allometry parameters for the ratio of the respiratory and cardiac rates (both in 1/s).

1.56 Use Table 1.14 to find the allometry parameters for the ratio of the volumes per breath (tidal volume) and per heart beat (heart stroke volume).

1.57 Use Fig. 1.21 to comment on whether the same mass-dependent-only allometric rules should be used within a species from birth to adulthood.

1.58 (a) Arm length scales as the body height to the 1.0 power for people older than 9 months and to the 1.2 power for those younger. At 9 months of age, a male is 61 cm tall and has an arm length of 23 cm. When that male was 0.42 yr old he had a height of 30 cm and when he will be 25.75 yr old he will have a height of 190 cm. What would the arm length be expected to be at these earlier and later times, alternatively using the age-correct and age-incorrect scaling exponents?

(b) Is this an example of allometric or isometric scaling?

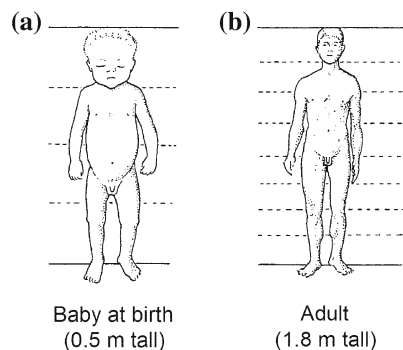
1.59 Scaling arguments can also be used to understand some general trends.

(a) If the linear dimension of an object is L , show that its surface area varies as L^2 , its volume as L^3 , and its surface to volume ratio as $1/L$, by using a sphere (diameter L) and a cube (length L) as examples.

(b) An animal loses heat by loss at the surface, so its rate of losing heat varies as its surface area, whereas its metabolic rate varies as its volume. In cold environments this loss of heat can be devastating. Do scaling arguments suggest animals would be bigger or smaller in cold climates?

(c) A cell receives oxygen and nutrients across its membrane to supply the entire volume of the cell. Do scaling arguments suggest that limitations in supplying oxygen and nutrients place a lower limit or upper limit in the size of cells? Why?

Fig. 1.21 Human development, showing the change in body shape from birth to adulthood, for Problem 1.57 (from [32])



1.60 The strength of bones varies as their cross-sectional area, as we will see in Chap. 4. We have seen that this suggests how the diameter of a long bone scales with its length.

(a) Does this scaling relation mean that smaller creatures have thinner or thicker bones than bigger creatures assuming the same strength criterion?

(b) Does this “static” argument imply a limitation on how small or how large an animal can be?

1.61 (a) The work an animal of dimension L needs to propel itself a distance equal to its dimension is the needed force—which is proportional to its mass—times its dimension. Show this work scales as L^4 .

(b) This force must be supplied by muscles, and the work done by the muscles is this force times the distance the muscles can contract; this distance scales as the length of the muscles, which in turn scales with L . In Chap. 5 we will see that the force exerted by a muscle is proportional to its cross-sectional area. If the lateral dimension of the muscle also scales as L , show that the maximum work that can be done by the muscle scales as L^3 .

(c) For work done by muscles to scale as fast as that needed for locomotion, how must the lateral dimensions of muscles vary?

(d) Do these “dynamic” arguments limit how small or how large an animal can be [24]?

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Chapter 2

Statics of the Body

Abstract The statics of the body are investigated by first reviewing forces, torques, and equilibrium, and then applying these conditions to statics in a plane and a lever. Specific examples of statics in the body are analyzed, including that of the lower arm, hips, shoulder, knee, ankle, jaws, and lower back. Some of these are related to injuries in the body, such as those in the hips and lower back. Several of the discussions in this chapter form the basis for analyzing motion in the next chapter.

The study of the force balance of an object at rest is called “statics.” Moreover, the study of very slow motion can usually be treated as a series of static conditions—as if there were no motion; this is called “quasistatics.” After reviewing the conditions for static equilibrium in three dimensions, we will examine the useful simplification to two dimensions, examples of which can often be characterized as one of the three types of levers. We will then apply these equilibrium conditions to the lower arm, hip, and the spine (lower back). Statics is one important area in biomechanics [15, 24, 28, 38].

2.1 Review of Forces, Torques, and Equilibrium

Each force $\mathbf{F}$ can be resolved into components in the x , y , and z directions (F_x, F_y, F_z). In a static condition the sum of the forces $\mathbf{F}$ in each the x , y , and z directions is zero:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0. \quad (2.1)$$

The speed of the center of mass of the object in each direction is then constant, and will usually be assumed to be zero here. These forces can be in balance either for the entire body or for any part of the body.

Similarly, each torque τ can be resolved into components in the x , y , and z directions (τ_x , τ_y , τ_z). In a static condition, the torques τ about the x -, y -, and z -axes also each sum to zero for the entire body and for any body part:

$$\sum \tau_x = 0, \quad \sum \tau_y = 0, \quad \sum \tau_z = 0. \quad (2.2)$$

The speed of angular rotation of the object about each axis is then constant, and will usually be assumed to be zero here.

What actually is a torque? Forces describe changes in linear motion—which means changes in velocities, while torques describe how these same forces can change angular motion—which means changes in angular velocities. The diagram in Fig. 2.1b shows that a force $\mathbf{F}$ applied in the positive y direction (with component F) a distance $+r$ from the z -axis, leads to a torque about the z -axis τ_y of magnitude rF . This leads to motion in the counterclockwise direction, caused by an angular acceleration that increases the angle θ . This is defined as a positive torque about this axis. A negative torque would occur, for example, if the force were applied in the negative y direction. This would lead to motion in the clockwise direction, caused by an angular acceleration that decreases the (signed) angle θ .

In general, the torque (vector τ) about any axis is defined as the vector cross product between the distance vector from that axis to the point where the force is applied $\mathbf{r}$ and the force vector $\mathbf{F}$ (Fig. 2.1e)

$$\tau = \mathbf{r} \times \mathbf{F}. \quad (2.3)$$

(You do not need to understand or use this vector cross product, just the results that are given below.)

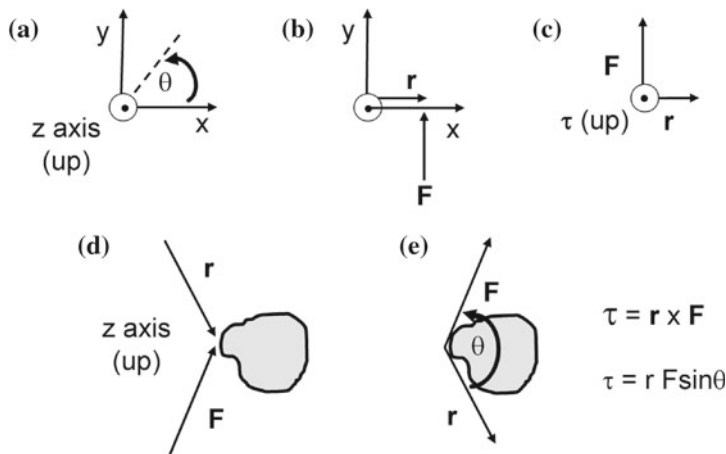


Fig. 2.1 Torques and relevant axes

Because vectors can be translated anywhere, things may be clearer if we move both $\mathbf{r}$ and $\mathbf{F}$, as in Fig. 2.1b, d, so they originate from where the force is applied, as in Fig. 2.1c, e, respectively. We will call the angle from the $\mathbf{r}$ vector to the $\mathbf{F}$ vector θ . The torque τ_z about the upward axis is

$$\tau_z = rF \sin \theta, \quad (2.4)$$

where r is the magnitude of vector $\mathbf{r}$ (the distance from the axis to the point where the force is applied) and F is the magnitude of vector $\mathbf{F}$. For $0^\circ < \theta < 180^\circ$ (or $0 < \theta < \pi$ in radians), $\sin \theta$ is positive and the torque is positive (Fig. 2.2a), while for $180^\circ < \theta < 360^\circ$ ($\pi < \theta < 2\pi$), $\sin \theta$ is negative and the torque is negative (Fig. 2.2b). When $\theta = 90^\circ (= \pi/2)$, $\sin \theta = 1$ and the torque is rF , as above. When $\mathbf{r}$ and $\mathbf{F}$ are either parallel ($\theta = 0^\circ (= 0)$) or antiparallel ($\theta = 180^\circ (= \pi)$), the torque is zero.

Clearly, only the component of the $\mathbf{r}$ normal to the $\mathbf{F}$, which we will call r' , contributes to the torque action. In fact, as Fig. 2.3 proves, $\tau_z = r'F$. Equivalently, only the component of $\mathbf{F}$ normal to $\mathbf{r}$, i.e., F' , contributes to the torque action, and $\tau_z = rF'$. As we will see, sometimes information is provided where these normal components of displacement or force are provided, and the torques can be calculated without explicitly determining the angle between the displacement and force vectors. Consequently,

$$\tau_z = rF \sin \theta = r'F = rF'. \quad (2.5)$$

This is true for any axis. The axis can be chosen cleverly for a particular problem to simplify analysis.

In linear motion, a force leads to an acceleration $\mathbf{a} = d^2\mathbf{r}/dt^2$, which is equivalent to a change in velocity $\mathbf{v} = d\mathbf{r}/dt$ (the magnitude of which is the speed v) or momentum $\mathbf{p} = m d\mathbf{v}/dt$, by

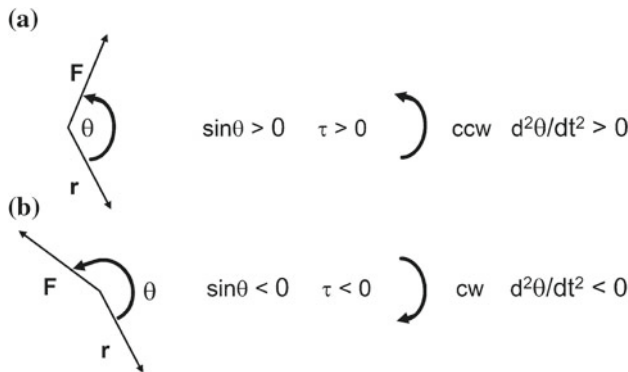


Fig. 2.2 Direction of torques, showing **a** positive and **b** negative torques

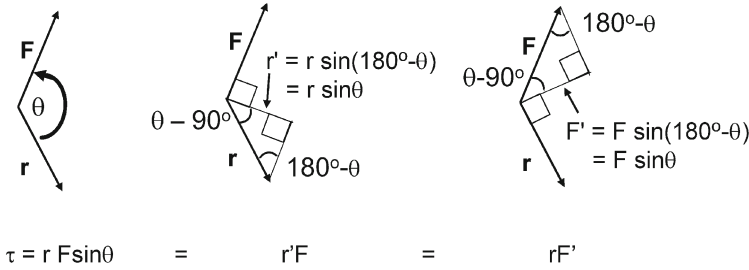


Fig. 2.3 Determining torques from using components of the displacement and force vectors that are normal to the force and displacement vectors, respectively

$$\mathbf{F} = m\mathbf{a} = m \frac{d\mathbf{v}}{dt} = \frac{d\mathbf{p}}{dt}, \quad (2.6)$$

where m is the mass of the object. Similarly, a torque leads to an analogous change in the angle θ and angular frequency $\Omega = d\theta/dt$, and orbital angular momentum L

$$\tau = I \frac{d\Omega}{dt} = \frac{dL}{dt}, \quad (2.7)$$

where I is the moment of inertia and $L = I\Omega$. In static situations the sum of the forces and torques on the object is zero, so the right-hand side of (2.7) equals zero.

By the way, what we are defining as torques are indeed commonly called “torques” in connection to the rotational and twisting motions of objects, as in this chapter and Chap. 3, but are instead called “moments” in connection to the bending of objects, as in Chap. 4.

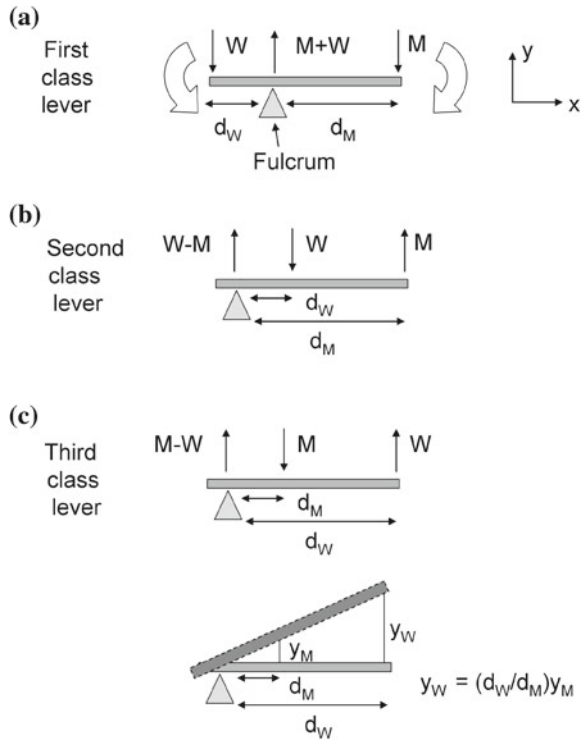
2.2 Statics: Motion in One Plane and Levers

Many problems involve motion in one plane, say the xy plane—for which z is a constant. For example, the motion of knees and elbows is in one plane. Some problems involving motion of the leg about the hip can be treated in these two dimensions. The six equations in (2.1) and (2.2) then reduce to three equations:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum \tau_z = 0. \quad (2.8)$$

(We will adopt this xyz coordinate system because it is conventionally used in two-dimensional problems, even though it differs from the coordinate system convention we adopted for the body in Fig. 1.1.)

Fig. 2.4 Three types of levers, **a** first, **b** second, and **c** third class levers. The large increase in the distance the weight moves over the change in muscle length in the third class lever is also seen in part (c)



These types of problems can be classified as one of the three types of levers (Fig. 2.4). There are examples in the body of each. They can be described by how a weight W and a force M , provided by a muscle, act on a solid object, say a bone resting on a fulcrum; this represents an articular joint. The weight can include that of parts of the body as well as external weights. The weight and muscle act at distances d_W and d_M from the joint. For each type of lever the total torque is zero when

$$Md_M = Wd_W, \quad (2.9)$$

so

$$M = \frac{d_W}{d_M} W. \quad (2.10)$$

The relative directions of the forces and the relative distances of the weight and muscle forces from the joint are different for each type of lever.

In a first class lever, the weight and muscle act on opposite sides of the fulcrum and are in the same direction (Fig. 2.4a). This is the least common type of lever in the body. Using the x, y coordinate system shown, there are clearly no forces in the x direction so the first equation in (2.8) is automatically satisfied. Since the weight and muscle both act in the same direction—downward—force balance in the y direction

requires that the fulcrum provides an upward force of $W + M$. Balancing torques in the z direction requires a choice of a z -axis. Any axis normal to the xy plane can be chosen. The simplest one is an axis at the fulcrum. The weight provides a torque of Wd_W , while the muscle provides a torque of $-Md_M$. The signs are consistent with the above discussion. The fulcrum provides no torque about this axis because the distance from the fulcrum to the axis is zero. So

$$\sum \tau_z = Wd_W - Md_M = 0. \quad (2.11)$$

This leads to (2.9), which tells how large the muscle force must be to maintain equilibrium. If the muscle cannot provide this large of a force, there can be no static condition. (Example: The lead ball is too heavy to hold up.) If the muscle provides more than this force, there is purposeful motion. (Example: The baseball is being thrown, as we will see later.) In these two cases, $\sum \tau_z$ in (2.11) is not zero.

It may seem that we cheated by choosing the axis at the fulcrum. Actually, we could have chosen the axis anyway in the xy plane. To prove this let us choose the axis anywhere along the bone, say a distance x to the right of the weight (Fig. 2.5). The torques provided by the weight, fulcrum, and muscle are now Wx , $(W + M)(d_W - x)$, and $-M(d_W + d_M - x)$, respectively. Balance requires

$$\sum \tau_z = Wx + (W + M)(d_W - x) - M(d_W + d_M - x) = 0, \quad (2.12)$$

which reduces to (2.11) again.

One type of the first class lever is a seesaw or teeter totter. A second type is the head atop the spinal cord, where the weight of the head is balanced by the downward effective force of the muscles (Fig. 2.6a). In a third example, the triceps brachii pull on the ulna about the elbow pivot balanced by the forces on the forearm. With the upper arm down, the triceps brachii can balance an upward force pushing the hand up (Fig. 2.6b). (With the upper arm pointed up, the triceps brachii can also balance the hand holding a weight.) Because $d_M \ll d_W$ here, the force that the triceps brachii needs to exert is much greater than the forces exerted at the hand.

In a second class lever, the muscle and weight act on the same side of the fulcrum, and the weight is nearer to the fulcrum, so $M < W$ (Fig. 2.4b). The second class lever has a mechanical advantage, which means that the *mechanical advantage*, defined as $W/M = d_M/d_W$, is > 1 . This type of lever is the second most common in the

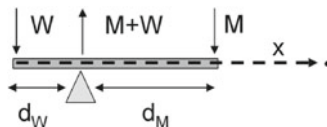
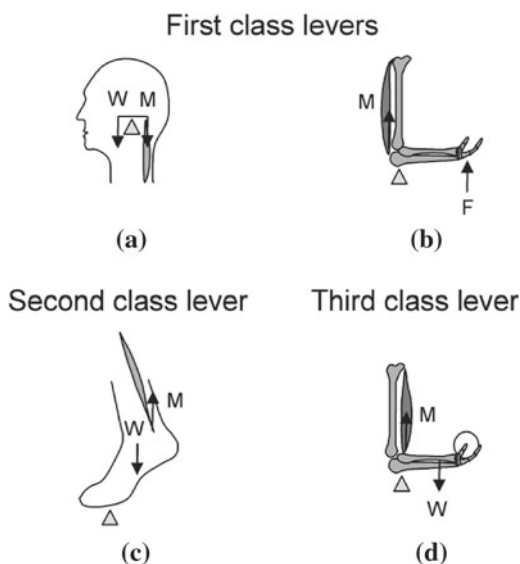


Fig. 2.5 Displacing the axis for calculating torques to the *right* of the weight by a distance x , as shown for a first class lever. For the axis chosen at the fulcrum $x = d_W$. The axis can be laterally displaced anywhere, to the *left* or *right* (as shown) of the lever, above or below it, or in it

Fig. 2.6 Examples of first (a, b), second (c), and third (d) class levers in the body



body. One example is standing on tiptoes: the rotation of the foot about the toes (the fulcrum), which would be caused by the weight of the foot, is balanced by the muscle force transmitted by the Achilles tendon (Fig. 2.6c). Another example is pushing down with the triceps brachii.

In third class levers, the muscle and weight are again on the same side of the fulcrum, but now the muscle is nearer to the fulcrum than the weight (Fig. 2.4c). This is the most common example in the body. Because often $d_M \ll d_W$, we see that $M = (d_W/d_M)W \gg W$. This arrangement means that very large forces must be exerted by the muscles because of this d_W/d_M amplification, which seems to be a big disadvantage (and is literally a mechanical disadvantage). However, something else is gained in this tradeoff in design. As seen in the Fig. 2.4c, when the bone rotates a given angle, causing a vertical displacement y_M at the muscle, there is an amplification of the distance traveled at the position of the weight by d_W/d_M . As we will see in Chap. 5, muscles are able to contract only a small fraction of their length—which amounts to at most several cm in many muscles. The length of the biceps is about 25 cm, and the maximum contraction is by ~ 7 –8 cm. With this amplification, the weight can now move much more than this. One example of a third class lever is the balancing of the lower arm by the biceps brachii inserted on the radius (Fig. 2.6d). Another is holding a weight with an outstretched arm.

2.3 Statics in the Body

We will examine the planar forces in the static equilibrium of the lower arm, at the hip, and in the back. In analyzing the lower arm, we will choose successively more complex and realistic models. We will see that the forces in the hip and back are quite large, much more than one would expect, and explains why people often have problems in these parts of the body, problems that can lead to hip replacements and life-long lower back pain [1, 8, 36]. The approach for these problems is the same. We consider all elements in one plane and examine the forces in the (as defined) x and y directions and the torque in the z direction. Some of the approaches of [3, 28] are followed.

2.3.1 The Lower Arm

We will examine the equilibrium of the forearm balanced by the contraction of the biceps brachii inserted on the forearm long bone called the radius; this is a continuation of the discussion of third class levers. The relevant bones are shown in Figs. 2.7 and 2.8. In equilibrium, the biceps brachii force counters the potential rotation about the elbow joint by the weight held in the hand (Fig. 2.9a). We will examine this example for different models, using Fig. 1.15 and Tables 1.6 and 1.7 to provide anthropometric information. The forearm is $0.146H$ long and the hand length is $0.108H$, where H is the body height, so the in the hand is about $(0.146 + 0.108/2)H = 0.2H$ from the pivot. (The ball is in the middle of the hand.)

Case 1

The biceps brachii insert about 4 cm from the pivot axis. Say there is a weight W_W held in the hand, which is $d_W = 36$ cm from the pivot. (With $H = 180$ cm, $0.2H = 36$ cm.) Therefore $M = (d_W/d_M)W_W = (36\text{ cm}/4\text{ cm})W_W = 9W_W$. So for a weight of 100 N the muscle must provide a force of 900 N for balance. Here N stands for the MKS/SI unit of newtons. Since $1\text{ N} \simeq 0.225\text{ lb}$, equivalently, a 22 lb weight is balanced by 200 lb of force exerted by the biceps brachii (Fig. 2.9b).

We have made several assumptions and approximations in this example without explicitly stating them. It is always good to start with simple models. It is equally important to understand exactly what assumptions and approximations are being made. Then, the model can be made more realistic. Here, we have assumed that the forearm and upper arm make a 90° angle. We have also neglected the mass of the forearm.

Case 2

Now let us improve the model by including the weight of the forearm W_F (Fig. 2.9c). This is $\approx 0.022W_b$, where W_b is the body weight (Table 1.7). For a 70 kg (700 N, 160 lb) person, this is $\approx 15\text{ N}$ (3.4 lb). We can treat the effect of the weight of the

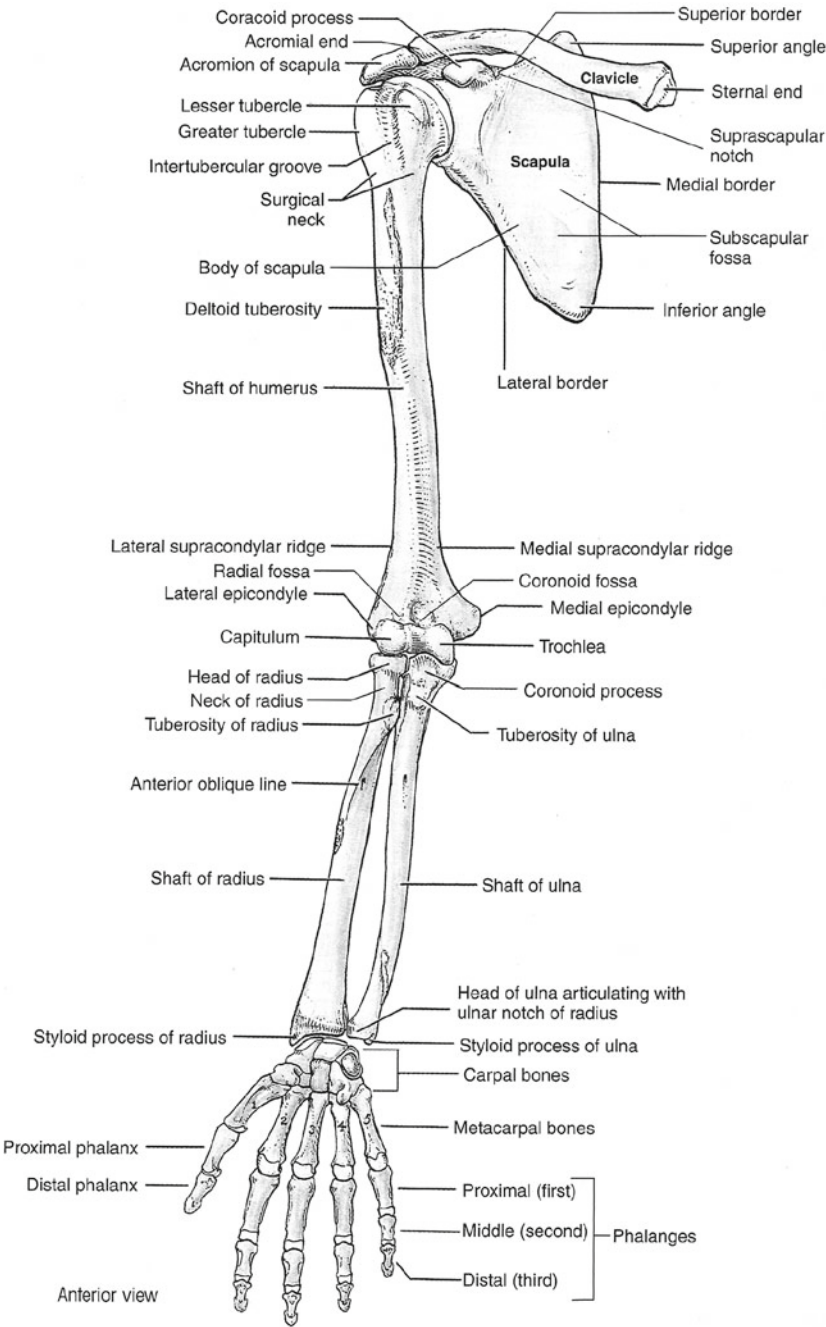


Fig. 2.7 Bones of the arm, anterior view. (From [20]. Used with permission)

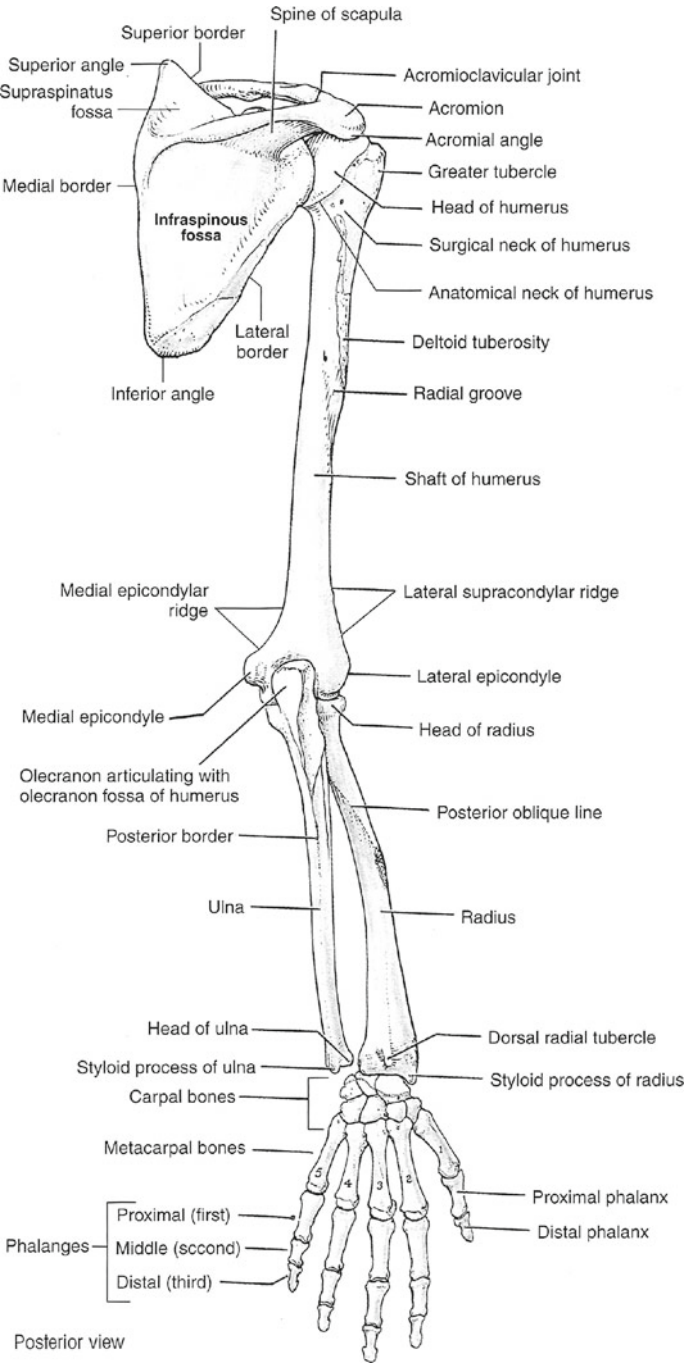


Fig. 2.8 Bones of the arm, posterior view. (From [20]. Used with permission)

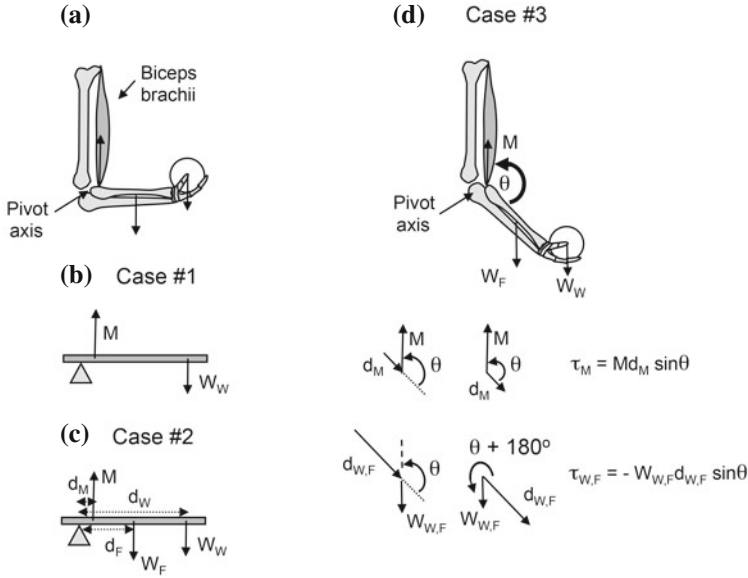


Fig. 2.9 a Weight held in the hand, showing the biceps brachii muscles. (b–d) Forces for the equilibrium of a weight held in the hand for Cases 1–3

forearm as if it were acting at its center of mass, which is approximately in the middle of the forearm, $d_F = 0.146H/2 = 13$ cm from the pivot:

$$\sum \tau_z = M d_M - W_W d_W - W_F d_F = 0 \quad (2.13)$$

$$M d_M = W_W d_W + W_F d_F \quad (2.14)$$

$$M = \frac{d_W}{d_M} W_W + \frac{d_F}{d_M} W_F. \quad (2.15)$$

The ratio d_F/d_M ($=13$ cm/ 4 cm), so now $M = 9 W_W + 3.25 W_F$ and the muscle force required to maintain equilibrium has increased to $900\text{ N} + (3.25)(15\text{ N}) = 950\text{ N}$ (210 lb).

Case 3

What happens if we no longer assume that the forearm and upper arm make a 90° angle? Let us keep the upper arm vertical and let the forearm make an angle θ , which can range over 142° (Table 1.10). The force due to the muscle is then still vertical, and those due to the weights of the forearm and ball are, of course, downward. From Fig. 2.9d we see that the torque caused by each of these three forces is multiplied by $\sin \theta$. Now

$$\sum \tau_z = M d_M \sin \theta - W_W d_W \sin \theta - W_F d_F \sin \theta = 0 \quad (2.16)$$

and we arrive at the same result that

$$Md_M = W_Wd_W + W_Fd_F. \quad (2.17)$$

Actually, we made additional assumptions in this example that we will re-examine later. The distance from the pivot where the biceps brachii insert on the radius really changes with θ (Fig. 3.46). Also, while this analysis suggests that the muscle force M required for equilibrium is the same for all angles, there is a subtlety in this result. Equation (2.16) gives the muscle force needed to maintain equilibrium. As we will see in Chap. 5, muscles can exert forces up to a maximum value. If the M from (2.17) can be achieved, then there can be equilibrium; if it cannot, then the static condition cannot be achieved. The maximum force that a muscle can exert depends on its length, which, from Fig. 2.9, is clearly a function of θ . So the M in (2.17) may be achievable at some angles (nearer 90° , where the maximum force turns out to be greatest) and not at others.

Case 4

The biceps brachii are not the only muscles used to flex the elbow. What happens if we also include the contributions of these other muscles? Fig. 2.10 shows that the biceps brachii, the brachialis, and the brachioradialis all contribute to this flexing. Assuming that $\theta = 90^\circ$ (which may not be a good assumption for each muscle), (2.13) is modified to

$$\sum \tau_z = M_1d_{M_1} + M_2d_{M_2} + M_3d_{M_3} - W_Wd_W - W_Fd_F = 0 \quad (2.18)$$

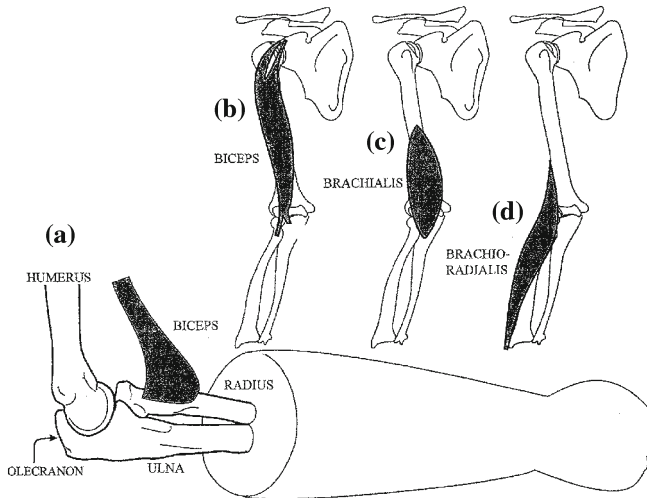


Fig. 2.10 Sketch of the elbow joint for analyzing the statics of the lower arm for Case 4, with the three muscles, the biceps (brachii), brachialis, and brachioradialis, shown in (b–d). (From [18])

$$M_1 d_{M_1} + M_2 d_{M_2} + M_3 d_{M_3} = W_W d_W + W_F d_F \quad (2.19)$$

where M_1 , M_2 , and M_3 represent the forces exerted by the three muscles M_i , respectively. If the physiological cross-sectional areas of the three muscles are A_1 , A_2 , and A_3 , respectively (which we usually call PCAs), and the muscle force for each can be assumed to be proportional to this area (which is a pretty good assumption), then $M_i = k A_i$, for $i = 1, 2, 3$. (We will see that this is a good assumption with k reaching a maximum of $\sim 40 \text{ N/cm}^2$ or so.) So,

$$k A_1 d_{M_1} + k A_2 d_{M_2} + k A_3 d_{M_3} = W_W d_W + W_F d_F \quad (2.20)$$

$$k = \frac{W_W d_W + W_F d_F}{A_1 d_{M_1} + A_2 d_{M_2} + A_3 d_{M_3}} \quad (2.21)$$

$$M_1 = k A_1 = A_1 \frac{W_W d_W + W_F d_F}{A_1 d_{M_1} + A_2 d_{M_2} + A_3 d_{M_3}} \quad (2.22)$$

$$M_2 = k A_2 = A_2 \frac{W_W d_W + W_F d_F}{A_1 d_{M_1} + A_2 d_{M_2} + A_3 d_{M_3}} \quad (2.23)$$

$$M_3 = k A_3 = A_3 \frac{W_W d_W + W_F d_F}{A_1 d_{M_1} + A_2 d_{M_2} + A_3 d_{M_3}}. \quad (2.24)$$

Using the parameters from Table 2.1, we get $M_1 = 262 \text{ N}$ (biceps), $M_2 = 399 \text{ N}$ (brachialis), and $M_3 = 85 \text{ N}$ (brachioradialis) when we generalize Case 2. This compares to the $M_1 = 696 \text{ N}$ that we would obtain for Case 2 with the biceps alone, using $d_1 = 4.6 \text{ cm}$ (instead of the 4 cm used before, which led to 800 N). The total muscle force is 746 N , which is greater than 696 N because the brachialis has a relatively small moment arm.

Life is a bit more complex than this result suggests because we assumed that k has the same value for each muscle. Really $M_i = k_i A_i$, and all the k_i 's need not be the same, as long as k_i is less than the maximum that can be exerted by the muscles. Unfortunately, if all the k_i s are not assumed to be equal, we do not have enough information to solve this problem uniquely as posed. The body may solve the indeterminate nature of this problem (with more variables than conditions) by

Table 2.1 Data for the three elbow muscles used for flexion

Muscle	Moment arm d_i (cm)	Physiological cross-section (PCA) (cm^2)
Biceps (muscle 1)	4.6	4.6
Brachialis (muscle 2)	3.4	7.0
Brachioradialis (muscle 3)	7.5	1.5

Using data from [18, 39]

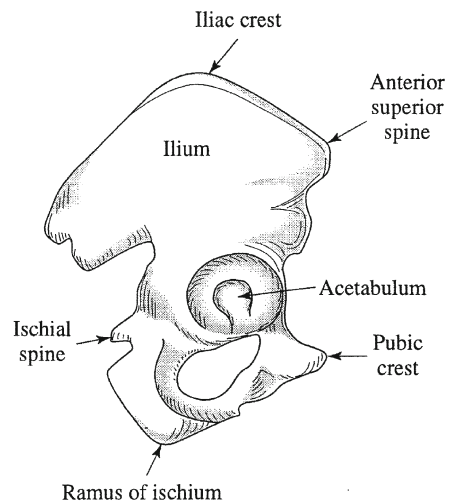
minimizing energy or optimizing the force distribution (shifting the load from one muscle to another) to rest specific muscles or by keeping the weight balanced better (so it will not tip in the hand).

2.3.2 Hip Problems

The hip (pelvis) is not a single bone, but several bones that are fused together (Figs. 2.11 and 2.12). The pelvis is composed of the pelvic girdle and two parts of the spinal cord, the sacrum and coccyx. The pelvic girdle itself is composed of the right and left coxal (or hip) bones. Each coxal (hip) bone is composed of three bones: the ilium (at the top = superior), pubis (bottom front = inferior, anterior), and ischium (iss-kee'-um) (bottom back = inferior, posterior). The acetabulum (a-si-tab-yoo'-lum) is the socket area where the femur of the leg (Fig. 2.13) is attached (“hip joint”). Actually, the head of the femur is in this socket, and is maintained there by the muscles attached at the greater trochanter. These muscles are collectively called the hip abductor muscles. (The hip abductor muscles are not the only one attached at the greater trochanter (see below), but they are the ones that contribute to the force needed for the equilibrium condition in this problem.)

First we will determine the force on the head of the femur and in the hip abductor muscles while the subject is standing on one leg, say the right leg. This is actually a good model for determining these forces during slow walking. The upper and lower leg and the foot are treated as a rigid body. This problem is solved in two steps. First, the forces in the whole body are analyzed and then the rigid leg is treated as a free rigid body, whose only interaction with the rest of the body will be the normal force from the hip [3, 27]. The bones in the leg are shown in Figs. 2.14 and 2.15.

Fig. 2.11 Right hip bone in adult. (From [3])



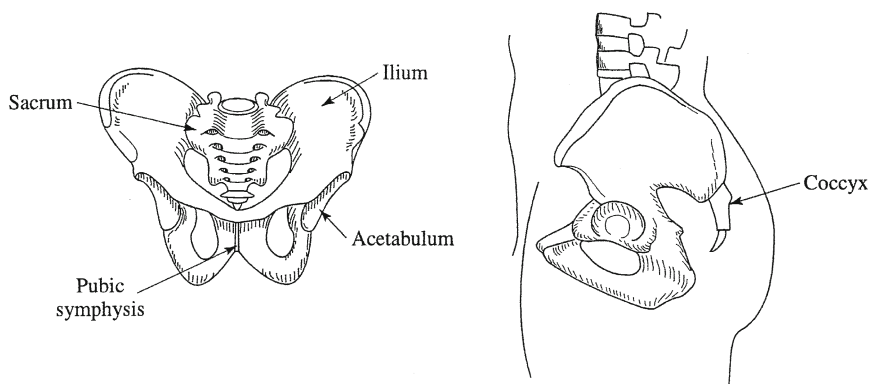
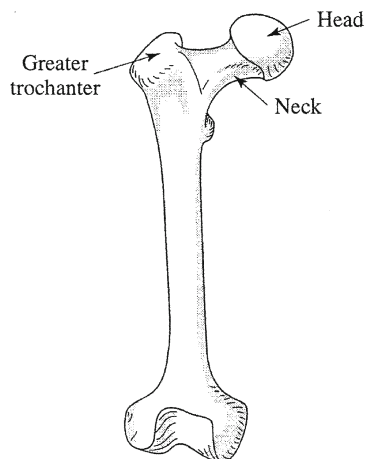


Fig. 2.12 Front and side views of the hip. (From [3])

Fig. 2.13 Anterior view of right femur. (From [3])



Total Body Equilibrium

There are only two forces on the body. The body weight W_b acts downward, and as if it all originated at the center of mass of the body, which is in the midline in the hip (Fig. 2.16). The foot feels an upward normal force from the floor of magnitude N . There are no forces in the x direction, and these two forces in the y direction must balance in equilibrium, so $N = W$. In equilibrium the body cannot start to rotate, so the torques are zero. It is clear from the Fig. 2.16 that this occurs when the foot is directly below the hip, in the midline. If we choose the pivot axis at the center of mass, the torque from the center of mass is zero because the distance term (from the axis to the center of mass) is zero and the torque from the normal force is zero

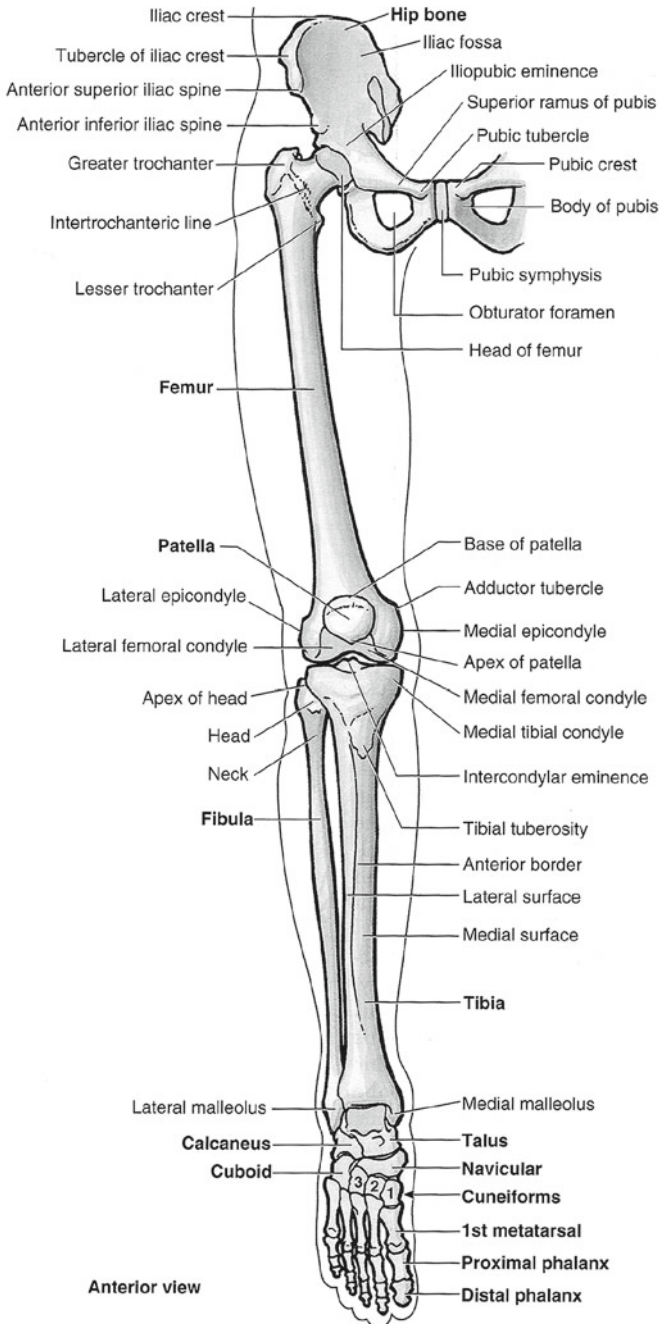


Fig. 2.14 Bones of the leg and hip, anterior view, with names of bones in bold. (From [21]. Used with permission)

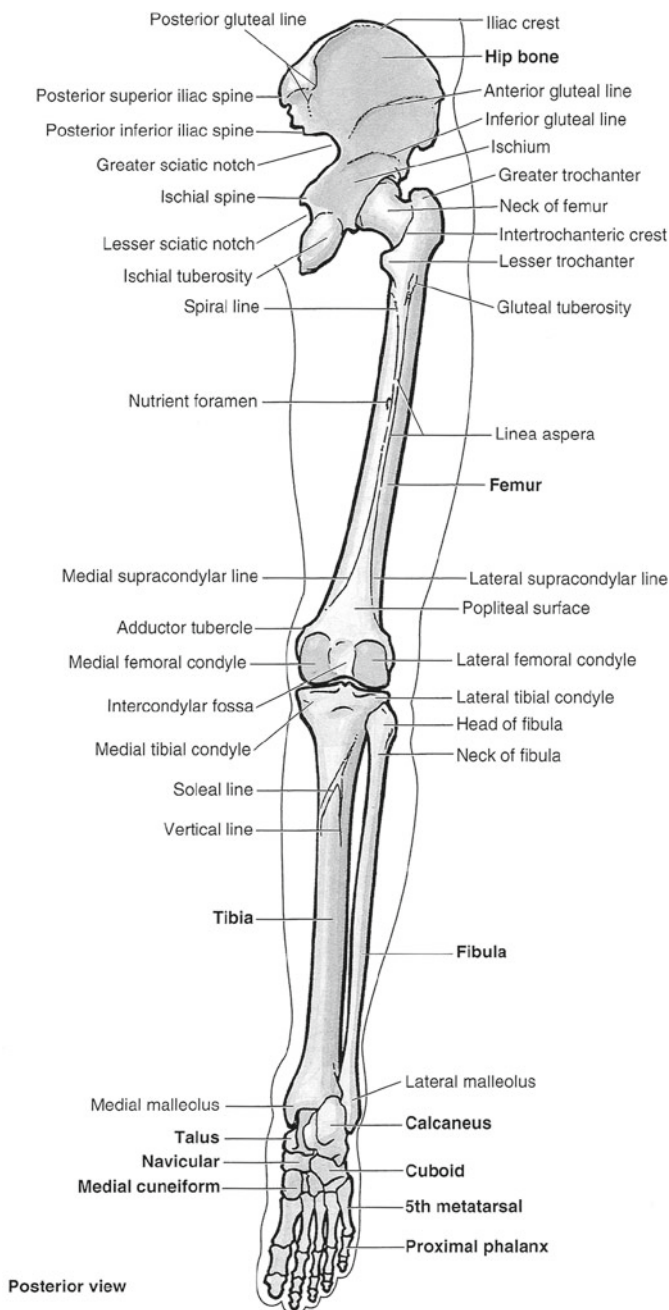
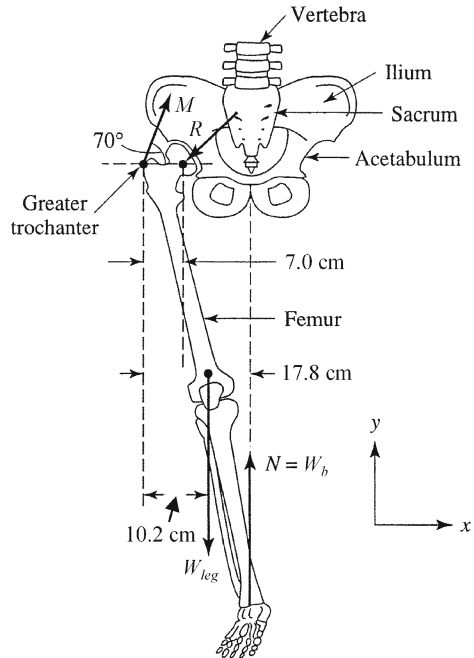


Fig. 2.15 Bones of the leg and hip, posterior view, with names of bones in bold. (From [21]. Used with permission)

Fig. 2.16 Anatomical diagram of the leg and hip for someone standing on one leg, or during slow walking, showing the forces on them and relevant dimensions, including the force exerted on the head of the femur by the acetabulum R and the net force exerted by the hip abductor muscles. (From [3])



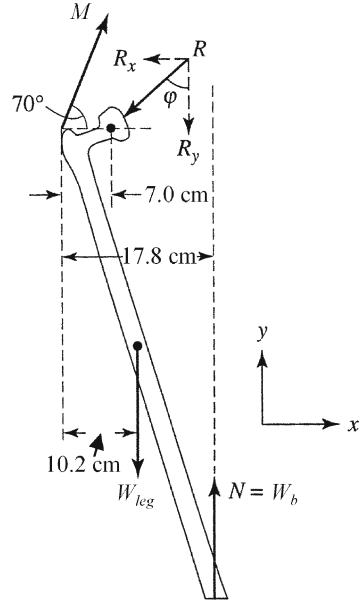
because the normal force is antiparallel to the distance vector ($\theta = 180^\circ$). (You can prove for yourself that the total torque is zero for any other axis normal to the xy plane.)

Equilibrium of the Individual Body Component

There are four external forces on the leg (Figs. 2.16 and 2.17):

- $\mathbf{N}$ is the normal force on the leg from the floor, and we know that $N = W_b$.
- W_{leg} is the weight of the leg. Table 1.7 shows that this is about $0.16W_b$. It acts as if it were applied at the center of mass of the leg, which is approximately halfway down the leg.
- $\mathbf{R}$ is the reaction force on the leg from the hip, and it is normal to the hip socket. We will define the x and y components R_x and R_y so they are positive (Fig. 2.17). Equivalently, we could define the magnitude of R and the angle it makes with the vertical, φ .
- $\mathbf{M}$ is the force (of magnitude M) due to the hip abductor muscles. There are actually three muscles involved here: the tensor fascia (fash-ee'-uh) latae (la-tuh) (see Fig. 3.3a), gluteus (gloo'-tee-us) minimus (see Fig. 3.3c), and the gluteus medius (see Fig. 3.3a, b). (The gluteus maximus muscle is what the author is sitting on as he is typing this.) These three muscles have a mass ratio of about 1:2:4 and, as we will see, this is roughly the ratio of the forces each can exert. The hip abductor muscle structure we consider is a composite of the three muscles. It has been shown that the effective action of this system is $\simeq 70^\circ$ to the horizontal, acting on the greater trochanter.

Fig. 2.17 Force diagram for a leg for someone standing on one foot, using Fig. 2.16. (From [3])



We have three equations with three unknowns: R_x , R_y , and M . The two force equations are

$$\sum F_x = M \cos 70^\circ - R_x = 0, \quad (2.25)$$

$$\sum F_y = M \sin 70^\circ - R_y - 0.16W_b + W_b = 0, \quad (2.26)$$

where W_{leg} has been replaced by $0.16W_b$.

We will choose the rotation axis to emanate from the center of the head of the femur because the reaction force from the acetabulum passes through this point. This makes the analysis easier, but, of course, the solution would be the same if we chose any other parallel axis. The relevant distances of interest are shown in the diagram (obtained from anatomical dimensions and geometry), as needed for torque analysis.

- The component of the distance vector perpendicular to the normal force (r') is 10.8 cm, so the normal force causes a torque of $(10.8 \text{ cm})W_b$. This is a positive torque because the normal force induces a counter clockwise rotation about the chosen z -axis (see Fig. 2.2).
- The component of the distance vector normal to the force of the weight of the leg is 3.2 cm and this force tends to induce a clockwise rotation, so it contributes a torque of $-(3.2 \text{ cm})W_{\text{leg}} = -(3.2 \text{ cm})(0.16W_b) = -(0.5 \text{ cm})W_b$.
- With the choice of the axis, the torque from the reaction force from the hip is zero, because the distance vector and normal force are antiparallel.